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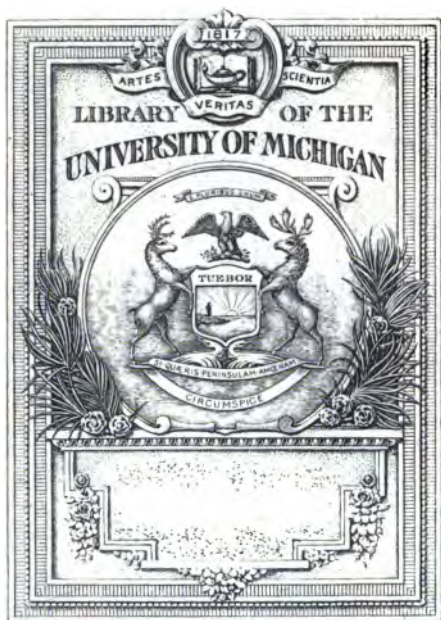
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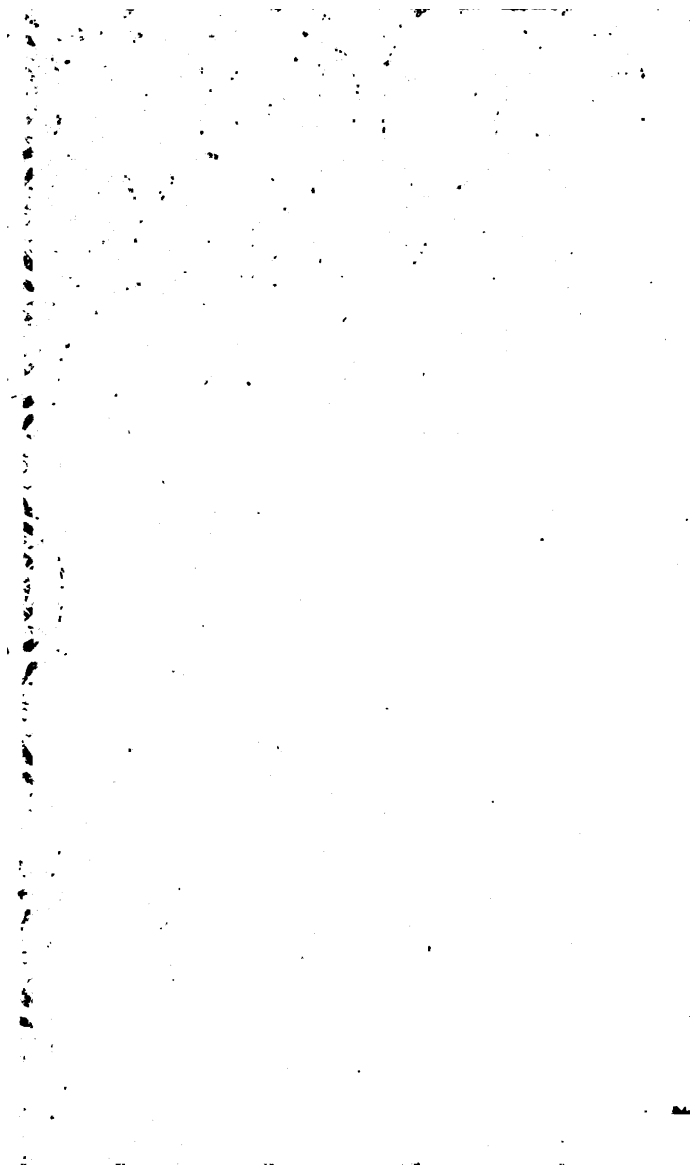
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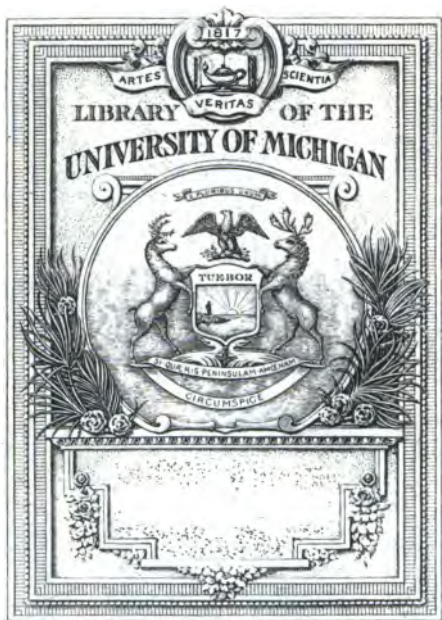
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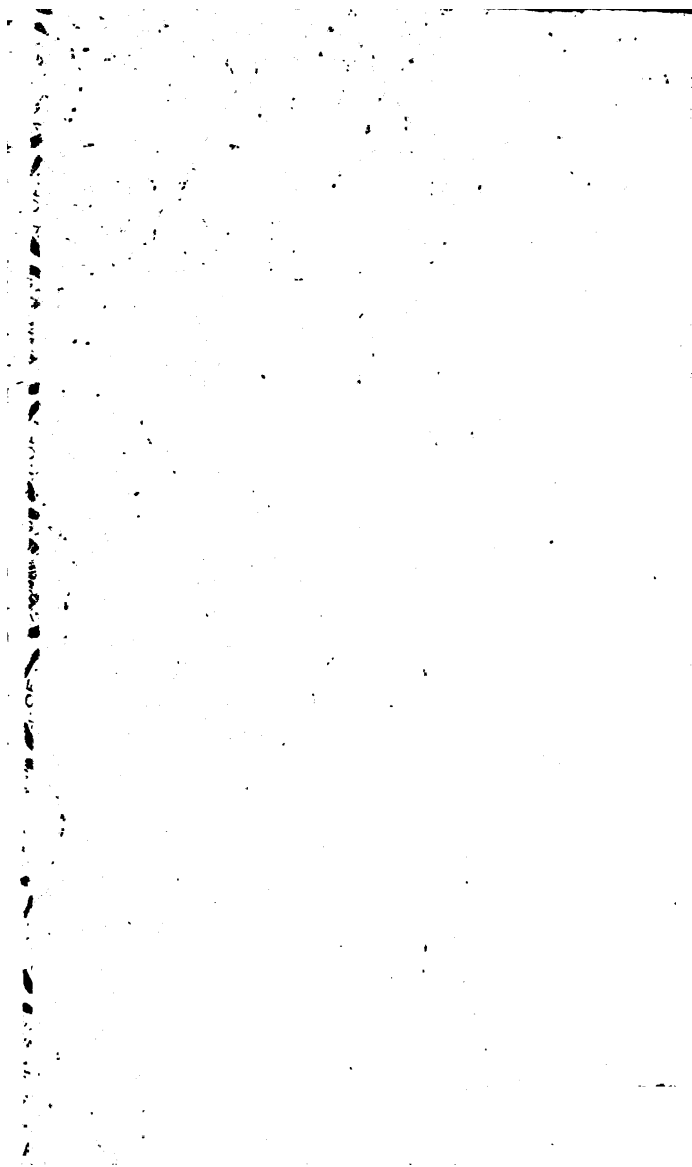
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CLAVIS USURÆ;
Or, A KEY to
INTEREST,
BOTH
Simple and Compound:

CONTAINING

Practical Rules, plainly express'd in *Words at length*; whereby all the various *Cases of Interest*, and *Annuities*, or *Leases*, either in *Possession*, or *Reversion*, and *Purchasing Free-hold Estates*, &c. may very easily be Resolv'd, both by the *Pen* and a small *Table of Logarithms*, hereunto annex'd; For all *Rates of Interest*, and *Times of Payment* whatsoever; Illustrated by *Variety of Examples*.

To which is Added,

Rules to be Observ'd in Estimating the *Value of Annuities*, or *Leases*, and *Insurances for Lives*, &c.

A L S O,

The *Business of Rebate or Discompt*, and the *Equation of Payments* (very useful for *Merchants* and other *Deallors*) is here *Rectified* and truly *Determin'd*.

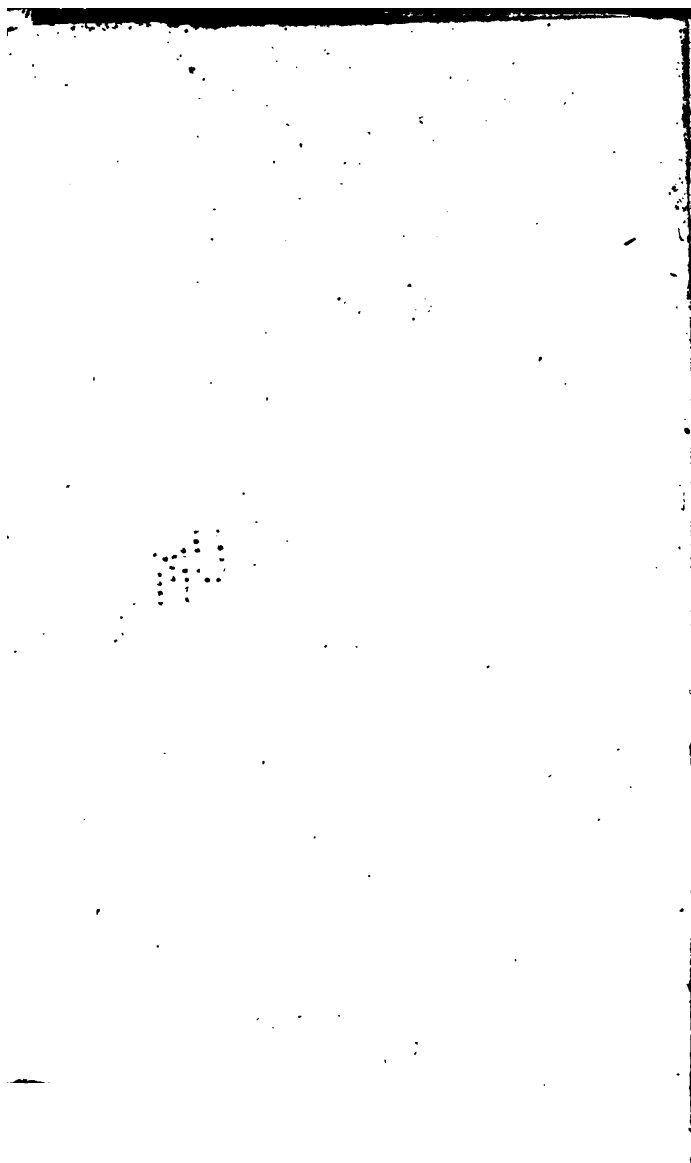
By J. WARD, *Author of the Young Mathematician's Guide*, &c.

THE SECOND EDITION.

L O N D O N:

Printed for J. CECILL in *Exeter Exchange* in the *Strand*; and Sold by H. CHAPELLE in *Grosvenor-Street* near *Grosvenor-Square*. 1740.

(Price 2 s.)



109-6-29 H.C.M.
To the Honourable
Sir John Wentworth,
OF
NORTH-ELMS-HALL,
IN THE
West-Riding of Yorkshire,
BARONET;

This TRACT, as an Acknow-
ledgment of Great Favours
and Obligations receiv'd,

Is most humbly Dedicated,

and Presented,

By

J. W.

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THE

PREFACE.

IT may be here expected that I should (according to the Custom of Prefaces) give the Reader some Account of the ensuing Pages, and of the Motives which induced me to Publish them; more especially, because there is already so many Books which Treat upon the same Subject.

'Tis true indeed, the Business of Interest and Annuities, &c. has been handled by several Authors; And I, amongst the rest, did adventure to Cast in my Mite upon that Subject, in a Compendium of Algebra, Published Anno 1695. which appears to be so well Approved of, that Doctor John Harris hath Transcribed and Inserted that whole Discourse into his *Lexicon Technicum*; Or, The Universal English Dictionary. Since then I wrote somewhat fuller upon the same Subject, in the *Algebraick Part* of the *Young Mathematician's Guide*, Published Anno 1709.

But even that, and all that I've hitherto seen upon this Subject, seems to fall short of what might be desired: For most Authors, which treat of Interest and Annuities, &c. do perform their Computations by the Help of particular Tables, Calculated to several Rates of Interest and Discount.

And those Authors which perform their Computations in Compound Interest, &c. by Logarithms, seem (to me) to be too general and short in their Theorems, by which those Calculations are perform'd, especially in respect of the different Times of Payment, &c.

And what I've heretofore done has been by Algebraick Theorems, to be perform'd either by the Pen, or with the help of a small Table, Calculated on purpose; which was indeed rather to shew what may possibly be perform'd by the Pen (without a great many Tables of several Rates)

The P R E F A C E.

than for common Practice : Nor can it be supposed, that every Reader, which may have Occasion to peruse the Business of Interest, is fitly qualified to understand those Theorems so well, as to apply them to Practice.

And for that Reason I was requested by an ingenious Gentleman (to whom I stand highly oblig'd) to Explain those Theorems, and Reduce them into Practical Rules, express'd in Words at length, that so they might become of general Use; the which I've accordingly done along with the Theorems, so plainly, that (I presume) any One, who is but a little vers'd in the first Four Rules of Arithmetick, may be able to Resolve all the various Cases relating to Interest, both Simple and Compound, with those of Annuities, or Leases, &c. for any proposed Time, and at any given Rate of Interest, both by the Pen, and a small Table of Logarithms herewith annex'd. Also I've added a New Method of finding the true Equated Time, for the Discharging of several Sums at one certain Payment, without Loss either to the Debtor, or Creditor, a thing which hath hitherto been imperfect.

And in order to render the whole Work as plain and easy as the subject could at possibly I can, I've given Two or Three Examples in every Case, for different Rates of Interest, and Times of Payments, with their Operations as large as the Rules are needful so well express'd in Words, (according to the Author's Sense) yet there may (and perhaps will) arise some seeming Difficulties in them, which Examples will help to Explain and render Easy.

To be brief; When this Tract was perus'd in Manuscript by a Person who has very good Skill in the Business of Interest, &c. it gain'd such ample Satisfaction, that I was prevail'd with to Publish it; and accordingly I've adventured to send it Abroad into the World, in hopes it will be found useful, which is the chief thing desir'd.

London, Novemb.
21th, 1709.

By

J. W A R D.

THE CONTENTS.

CHAP. I. Of Decimal Arithmetick.

Sect.	Pag.
1. N otation of Decimals,	1
2. Addition and Subtraction of Decimals,	3
3. Multiplication of Decimals,	4
4. Division of Decimals,	5
5. Reduction of Vulgar Fractions to Decimals,	7

CHAP. II. Of Logarithms.

1. The Definition of Logarithms, &c,	10
2. To find the Logarithm of any given Number,	12
3. To find the Number to any given Logarithm,	14
4. To perform Multiplication by Logarithms,	15
5. Division perform'd by Logarithms,	17
6. To Extract the Roots by Logarithms,	18

CHAP. III. The Calculation of Questions. in Simple Interest, and Annuities.

1. Of Simple Interest, &c.	22
2. Of Annuities or Pensions, &c. in Arrears,	32
3. To find the Present Worth of Annuities,	39

CHAP.

The CONTENTS.

CHAP. IV. The Calculation of Questions in Compound Interest, and Annuities, &c.

Sect.	Pag.
1. Of Money Lett-out at Compound Interest,	53
2. Of Annuities, or Rents in Arrears,	57
3. To find the Present Worth of Annuities,	73
4. Of Annuities, or Leases, &c. in Reversion,	93

CHAP. V. Of Purchasing Free-hold Estates, and how to Estimate the Value of Annuities and Leases for Lives, &c.

1. Of Purchasing Free-hold Estates,	99
2. How to Estimate Annuities &c. for Lives,	104

CHAP. VI. Of Rebate and Discompt, and the true Equated Time of several Pay- ment, either at Simple, or Compound Interest.

1. To find the Discompt at Simple Interest,	114
2. The Equated Time of Payments truly determin'd,	118

A Table of Logarithms, for Numbers In-
creasing in their proper Order : From 1,
To 10000.

A Postscript, concerning the Payment of
Simple Interest, in less Time than a
compleat Year.

A
KEY TO INTEREST,
 BOTH
Simple and Compound, &c.

C H A P. I.
 Of Decimal Arithmetick.

I Do here take it for granted, that the Reader is well acquainted with *Vulgar Arithmetick*, especially the common Rules; viz. *Addition, Subtraction, Multiplication, and Division of whole Numbers*: But because the Computations of *Interest*, and *Values of Annuities, &c.* cannot well be perform'd by *Logarithms*, or indeed by any other Method whatsoever, without some Knowledge of those Rules in *Decimal Fractions or Parts*, which perhaps he is not so well acquainted with; I therefore thought it convenient to give a brief Account thereof.

Sect. I. *Notation of Decimals.*

Any thing which is called One, (whether it be *Coin, Weight, Measure, or Time, &c.*) as one *Foot, one Yard, one Pound, one Shilling, one Year, or one Day, &c.* we conceive it to be *Divided* into Ten equal Parts, and every one of those Parts, are supposed to be sub-divided into other Ten equal Parts; and so on by a *Decimal Division* (viz. by Tens) *ad infinitum.*

4. But *Cyphers* prefixed before *Decimal Parts*, do decrease their *Value*, by removing them further from the *Comma*, or *Unit's* place.

Thus $\left\{ \begin{array}{ll} 0,5 & \text{is } 5 \text{ parts of } \textit{Ten}. \\ 0,05 & \text{is } 5 \text{ parts of a } \textit{Hundred}. \\ 0,005 & \text{is } 5 \text{ parts of a } \textit{Thousand}, \&c. \end{array} \right.$

Consequently the true *Value* of all *Decimal Parts* is easily known by their distance from the *Unit's* place (as above :) Which being well understood, the following *Rules* will be found very easie.

Sect. 3. Addition and Subtraction of Decimals.

Having set down all the proposed Numbers in their respective places; viz. every *Figure*, as well of the *Decimal Parts*, as of the whole Numbers, directly underneath those of the same *Value* (or Name;) which may be very easily done, if the *Comma's*, or separating *Points*; are carefully placed directly one under another.

Then $\left\{ \begin{array}{l} \text{Add, or Subtract them as if they were all} \\ \text{whole Numbers; and from their Sum, or} \\ \text{Difference, Cut off, by the separating Com-} \\ \text{ma, so many places of Decimal Parts, as} \\ \text{there are in any of the given Numbers.} \end{array} \right.$

Examples in Addition.

Let it be required to find the *Sum* of 43,25 and 2,45 and 29,9 and 7,054 These Numbers being rightly set down will stand

Thus $\begin{array}{r} 43,25 \\ 2,45 \\ 29,9 \\ 7,054 \\ \hline \text{Sum } 82,654 \end{array}$

Again $\begin{array}{r} 378,0009 \\ 59,025 \\ 470,9 \\ 92,0741 \\ \hline \text{Sum } 1000,0000 \end{array} \left. \vphantom{\begin{array}{r} 378,0009 \\ 59,025 \\ 470,9 \\ 92,0741 \end{array}} \right\} \text{Add}$

Examples in Subtraction.

From 753,24	From 75,094	From 543,000
Take 469,25	Take 68,596	Take 267,546
Rem. 284,00	6,498	275,454

Note, Addition and Subtraction do mutually prove each other, as in whole Numbers; of which I suppose it needless to insert Examples.

Sect. 3. Multiplication of Decimals.

Whether the Factors (viz. the Numbers) proposed to be Multiplied together, are either all Decimal Parts, or Decimals joyned to whole Numbers, Multiply them together as if they were all whole Numbers, and for the true Value of their Product, observe this Rule.

Rule { Cut off (viz. separate) with a Comma, so many places of Decimal parts in the Product as there are in both the Factors counted together.

Examples.

Suppose it were requir'd to Multiply 21,3 with 4,56

$$\begin{array}{r} \text{The given Factors } \left\{ \begin{array}{l} 21,3 \\ 4,56 \end{array} \right. \\ \hline 1278 \\ 1065 \\ 852 \\ \hline \end{array}$$

Product 97,128

$$\text{Or } \left\{ \begin{array}{l} 4,56 \\ 21,3 \end{array} \right. \\ \hline 1368 \\ 456 \\ 912 \\ \hline$$

97,128

Again, Let it be requir'd to Multiply 4573 into 7,546: And 36,4078 into 549,3 The Work may stand

$$\begin{array}{r} \text{Thus } \left\{ \begin{array}{l} 4573 \\ 7,546 \end{array} \right. \\ \hline 27438 \\ 18292 \\ 22865 \\ 32011 \\ \hline 34507,858 \end{array}$$

$$\left. \begin{array}{l} \text{Multiply } \left\{ \begin{array}{l} 36,4078 \\ 549,3 \end{array} \right. \\ \hline 1092234 \\ 3276702 \\ 1456312 \\ 1820390 \\ \hline 19998,80454 \end{array} \right\} \text{Products}$$

Note,

Decimal Arithmetick.

5

Note, It sometimes happens, that in *Multiplying* Parts with Parts only, there will not be so many *Figures* in their *Product* as there ought to by places of Parts by the *Rule* : In that Case you must supply the Defect by *prefixing Cyphers* to their *Product* ; As in these following

Examples.

$\begin{array}{r} 0,2365 \\ 0,2435 \\ \hline 11825 \\ 7095 \\ 9460 \\ 4730 \\ \hline 0,05758775 \end{array}$	}	<i>Multiply</i>	{	$\begin{array}{r} 0,03472 \\ 0,02364 \\ \hline 13888 \\ 20832 \\ 10416 \\ 6944 \\ \hline 0,0008207808 \end{array}$
		<i>Products</i>		

Sect. 4. Division of Decimals.

Division of *Decimals* is perform'd. in all respects like that of whole *Numbers* ; And for discovering the true *Value* of the *Quotient*, observe this *General Rule*.

Rule { The places of *Decimal* parts in the *Divisor* and *Quotient* being counted together, must always be equal in Number with those in the *Dividend*.

This *General Rule* admits of Four *Cases*.

Case 1. When the Places of *Decimal* Parts in the *Divisor* and *Dividend* are equal in Number, then the *Quotient* will be all whole *Numbers* ; As in the Two following *Examples*.

<i>Dividend.</i>	
7140,38	(947 the <i>Quotient</i> .)
6786	
3543	
3016	
5278	
5278	
603	

B 3

Again.

6. Decimal Arithmetick.

Again, If it were required to Divide 2,6925 by 0,0075

Dividend.
Divisor 0,0075) 2,6925 (359 *Quotient.*

Case 2. When the Number of places of Decimal parts in the Dividend, exceed those in the Divisor, Cut off the Excess for Decimal parts in the Quotient.

Examples.

7,54) 71,4038 (9,47 And 0,75) 2,6925 (3,59

Case 3. When there is not so many places of Decimal parts in the Dividend, as there is in the Divisor, then annex Cyphers to the Dividend, to make them equal, and the Quotient will be all whole Numbers ; As in *Case 1.*

Examples.

Let us suppose it were requir'd to Divide 1565,7 by 3,684 : And 3615 by 5,784 the Work must stand thus,

3,684) 1565,700 (425 And 5,784) 3615,000 (625

Case 4. If, after Division is finish'd, there are not so many Figures in the Quotient, as there ought to be Places of Decimal parts, by the General Rule, you must then supply their Defect, by prefixing Cyphers before the Quotient Figures.

Examples.

Let it be requir'd to Divide 3,5532 by 756

756) 3,5532 (47
 3024
 5292
 5292
 (0)

Here Division being finish'd, there are but *Two* Figures in the Quotient, whereas there ought to be Four places of Decimal parts, by the General Rule ; therefore two

Cyphers must be prefix'd before the Quotient Figures, thus 0,0047

That is, 756) 3,5532 (0,0047 is the true Quotient required. Suppose

Suppose it were required to Divide 0,0007475 by 0,575 ; viz. all Decimal parts ; Then It will be 0,575) 0,0007475 (0,0013 the Quotient, &c.

Note, Multiplication and Division do mutually prove the Truth of each other : That is, If in Multiplication you Divide the Product by either of the Factors, the Quotient will be the other Factor, if the Work be true.

So in *Division*, If you Multiply the Quotient and the Divisor together, their Product (*being Added to what remain'd after Division*) will be the Dividend, if that Work be true.

Sect. 5. To Reduce (or rather Change) Vulgar Fractions into Decimal Parts, and the contrary.

Rule { *Annex Cyphers to the Numerator of the given Fraction ; then Divide it by the Denominator, and the Quotient will be the Decimal parts sought.*

Examples.

Suppose it were requir'd to Reduce $\frac{3}{4}$ into Decimal parts : Then it will

be, 4) 3,00 (0,75 the Decimal parts required ; And $\frac{1}{2}$ is 0,5 Thus 2) 1,0 (0,5 the Decimal ; Or $\frac{1}{4}$ is 0,25 Thus 4) 1,00 (0,25 the Decimal.

And thus may the Decimal parts equivalent to any known part, or parts of *Coin, Weight, Measures, or Time, &c.* be easily found, if you first reduce the given parts of the *Coin, or Time, &c.* into a *Vulgar Fraction*, whose Denominator is the Number of those known parts contain'd in the Integer, and the given parts its Numerator.

Examples in Coin.

Suppose it were requir'd to find the Decimal parts of a Pound Sterling, equal to 17 s. 6 d.

First,

First, because 20 s. makes one Pound, therefore 17 s. is $\frac{17}{20}$ of a Pound; and because there's 240 Pence in a Pound, therefore 6 d. is $\frac{6}{240}$ of a Pound.

Then $20 \mid 17,00$ (0,85 is the Decimal of 17 s.

And $240 \mid 6,000$ (0,025 is the Decimal of 6 d.

Consequently their Sum 0,875 will be the Decimal parts equivalent to 17 s. 6 d. as was requir'd, &c.

Example in Time.

Let it be requir'd to find the Decimal parts equal to 126 Days.

First, because 365 Days do make a common Year, therefore 126 Days is $\frac{126}{365}$ parts of a Year.

Then $365 \mid 126,0000$ (0,3452 will be the Decimal parts equivalent to 126 Days, &c.

The like is to be understood in *Reducing*, or *Changing* the known parts of any proposed Integer into Decimal parts, equal to those known parts; or at least so near the Truth, as may be thought necessary to approach.

Now the contrary Operations to these, *viz.* to find the Value of any given Decimal parts of Coin, or Time, &c. is only the *Converse* of the Work above, and may be perform'd by this following Rule.

Rule { Multiply the given Decimal parts with such a Number of Units as are contain'd in the Integer to which the given Decimal does belong, and the Product will be the Value of the said Decimal.

Examples.

1. Let 25,7875 signifie 25 l. and the Decimal parts of 1 l. How many Shillings and Pence do these Decimal parts denote? That is, How many Shillings and Pence are equal to 0,7875 Decimal parts of a Pound Sterling.

First,

First, $25,7875$
 Multiplied with 20 the Shillings in 1 l.
 Gives Shillings $15,7500$ and the Decimals of 1 s.
 Multiply with 12 the Pence in 1 s.

$$\begin{array}{r} 150 \\ 75 \\ \hline \end{array}$$

Gives Pence $9,0000$ Hence the Answer is,
 That $25\text{ l. } 15\text{ s. } 9\text{ d.}$ is the same with $25,7875$
 Decimals of *Coin*.

2. Suppose the same $25,7875$ to signifie 25 Years,
 and the Decimal parts of one Year; How many *Days*,
Hours, &c. would those Decimal parts denote?

Answer, 25 Years, 287 Days, 10 Hours, and 30 Min.

Thus $25,7875$
 Multiplied with 365 the Days in one Year,

$$\begin{array}{r} 39375 \\ 47250 \\ 23625 \\ \hline \end{array}$$

Gives Days $287,4375$ and Decimal parts of 1 Day ,
 Multiply with 24 the Hours in 1 Day ,

$$\begin{array}{r} 17500 \\ 8750 \\ \hline \end{array}$$

 Gives Hours $10,5000$ and $0,5$ parts of 1 Hour ,
 Multiply with 60 the Minutes in 1 Hour .
 Minutes $30,0000$

And thus may any proposed Number of Decimal
 parts be *Reduced* or *Changed* into the known parts of
 what they represent, *viz.* whether they be parts of
Coin, *Weight*, *Measures*, or *Time*, &c. a due regard
 being had to the Number of Units which are contain'd
 in the Denomination of the thing to which the Deci-
 mal Parts belong.

C H A P. II.

The Definition and Use of Logarithms
in General.

Sect. I. Definitions.

Logarithms are a sort of Artificial Numbers, so adapted to correspond with Natural Numbers, that the *Addition* and *Subtraction* of them, do exactly answer to the *Multiplication* and *Division* of those Natural Numbers they are adapted to.

That is, If any two given Numbers are either to be *Multiplied*, or *Divided*, the Logarithms of those Numbers being accordingly *Added*, or *Subtracted*, their *Sum*, or *Difference* will be the Logarithm of that Natural Number, which is the *Product*, or *Quotient* of such *Multiplication*, or *Division*.

And that the Value of any *Product*, or *Quotient*, so found by the *Sum*, or *Difference*, of two Logarithms may be truly known, every Logarithm of the prime Numbers, *viz.* of 1 . 10 . 100 . 1000 &c. in whole Numbers ; And of 0,1 . 0,01 . 0,001 . 0,0001 &c. in Decimal parts, is the *Index* or *Characteristick* to all the intermediate Logarithms which are between them : As in this

T A B L E.

Whole Num.	Logarithms	Decim. parts	Logarithms
1	0.000000	1	0.000000
10	1.000000	0,1	9.000000
100	2.000000	0,01	8.000000
1000	3.000000	0,001	7.000000
10000	4.000000	0,0001	6.000000
&c.	&c.	&c.	&c.

This

This *Table* shews some of the principal *Logarithms*, or *Characteristicks*, both of whole Numbers, and of Decimal parts, with so many Cyphers annex'd to each of them (*like Decimal parts*) as the number of places are in the following *Table of Logarithms*: From whence it will be easie to perceive, That,

If 0, be made the Logarithm of 1. And 1, be made the Logarithm of 10. Then, all the intermediate Logarithms which belong to the natural Numbers, between 1, and 10, (*viz. the Logarithms of 2. 3. 4. 5. 6. &c.*) must needs be Decimals less than 1. And for the same reason, That,

If 1. be the Logarithm of 10. And 2. be made the Logarithm of 100. Then all the intermediate Logarithms, which belong to the natural Numbers between 10. and 100. will be a 1, with Decimal parts annexed to it: And consequently all the Logarithms of the natural Numbers between 100, and 1000, will be a 2, with Decimal parts annexed to it; and so on for higher Numbers, as in the *Table*.

Now all these principal Logarithms, *viz.* 0. 1. 2. 3. 4. &c. of whole Numbers, are call'd *Affirmative Indices*, or *Characteristicks*.

And those which belong to Decimal parts or *Fractions*, *viz.* $\overline{9}$. $\overline{8}$. $\overline{7}$. $\overline{6}$ &c. are call'd *Negative Indices* or *Characteristicks*; being distinguish'd from the Affirmative Indices by the Negative Sign — set over their Heads.

For, If 0, be the Logarithm of 1. (*as above*) Then it follows, that all the Logarithms of Fractions (*viz. of Numbers less than 1*) must needs be less than 0. That is, they must be *Negative Numbers*.

From what hath been here said, it will be easie to deduce the General Rule, by which the Distance of the natural Numbers from the Unit's place is always known.

Viz.

N. Viz. { That every Index or Characteristick is less by an Unit or 1. than the Number of places of Figures in the natural Numbers to which it belongs.

As for instance in the following Numbers, wherein I suppose 5381 to be a whole Number; whose Logarithm is 3.730863. Now, if this Natural Number 5381. be otherwise taken or varied in its places, then the Logarithms will stand thus:

Natural Numbers	Logarithms.
5381 —————	3.730863
538,1 —————	2.730863
53,81 —————	1.730863
5,381 —————	0.730863
0,5381 ———	9.730863
0,05381 —	8.730863
0,005381 —	7.730863 &c.

In these Examples you see, that the Logarithms are all the same, save only the *Indices* or *Characteristicks*, are alter'd according to the Distance of the first Figure of the natural Number from the Unit's place; which being once well understood, it will be easie to find the Logarithm of any given Number in the *Table of Logarithms*, and to prefix its proper *Index* to it.

Or, if any Logarithm, with its Index be given; To find its correspondent Number, so far as the *Table of Logarithms* extends; and upon occasion to one or two places further, with a very small trouble.

Seet. 2. To find the Logarithm of any given Number.

The first Page of the annex'd *Table* of *Logarithms* contains all the Natural Numbers in their proper Order from 1. to 100. And against every one of those Numbers is placed its *Logarithm*, with its *Index* or *Characteristick* before it.

Thus, against the Number 28 is its *Log.* 1.447158
And against the Number 89 is its *Log.* 1.949390.
And so on for the rest. In

In the first Column of all, the following Pages (*under Num.*) the natural Numbers proceed in their due Order from 100. To 1000. And in the next Column (*under 0*) against every one of those Numbers, is the Decimal Part of its Logarithm, without any Index; to which you must prefix its proper Index, according as the natural Number you make use of requires.

As for instance; Against the Number 856 (*under 0*) is 932474 To which if 2 the Index of 856 be prefix'd, it will become 2.932474 the compleated Logarithm of 856.

The other five Columns of each Page contains the Logarithms of all the Numbers from 1000 To 10000 Those in the Left Hand Pages are distinguish'd on the Top of the Columns with the Figures 0 . 1 . 2 . 3 . 4 . And those in the Right Hand Pages with 5 . 6 . 7 . 8 . 9. So that to find the Logarithm of any Number between 1000 and 10000 As suppose of 5468, you must look for the Three first Figures, *viz.* 546 in the first Column under *Num.* and for the last Figure, *viz.* 8 at the Top: Then in the Column under the last Figure 8 and right over-against the Three first Figures 546 there is 737829 To which if 3 the Index of 5468 be prefixed, you will then have 3.737829 the compleated Logarithm of 5468 as was required: And so for any other Logarithm of any proposed Number not exceeding 10000.

But if the proposed Number be above 10000 (*which is the Limits of the annexed Table*) Then the Logarithm of that Number must be found by help of the Common Difference of the Logarithms, which is in the last Column of every Page under *Diff.*

Thus,

Find the Logarithm of the first Four Figures of the given Number, without its Index, (as above) and Multiply the Common Difference which stands against that Logarithm (*under Diff.*) with the other Figures of the given Number,

Number, Casting off so many Figures of that Product as there are in the Multiplier.

Then Add the Remaining Figures of that Product, to the Logarithm of the first Four Figures, and to their Sum prefix the proper Index, and you will have the compleated Logarithm required.

Example.

Suppose it were required to find the Logarithm of 698476. First the Logarithm of 6984 is found in the Table (*as above*) to be 844104, and against it, under *Diff.* is 62. This 62 being Multiplied with 76 (*the other Two Figures of the given Number*) produces 4712. Cut off the 12 (*viz. the Two last Figures*) and then Add the 47. to the Logarithm last found, and the Sum will be 844151; to which prefixing 5 the proper Index of the given Number 698476, it will be 5.844151 the Logarithm of 698476 as was required.

Sect. 3. To find the Number to any given Logarithm.

Omit the Index or Characteristick, of the given Logarithm, and then seek it in the Table of Logarithms; and if it can be exactly found there, then the Number in the first Column (*under Num.*) with that on the Top over the Logarithm, will be the Number required. But if the given Logarithm (*without its Index*) cannot be exactly found in the Table, then the proper Number agreeing to that Logarithm, may be found by the help of the Common Difference of the Logarithms.

Thus, $\left\{ \begin{array}{l} \text{From the given Logarithm Substract the next} \\ \text{Less, and to the Remainder Annex Cypbers; and} \\ \text{then Divide it by the Common Difference found} \\ \text{against the next less Logarithm, under Diff. and} \\ \text{the Quotient will be a Number that must be An-} \\ \text{nexed to the Number already found against the} \\ \text{next less Logarithm, according as the Index of} \\ \text{the given Logarithm denotes.} \end{array} \right.$

Example

Example.

Suppose 5.660279 were a given Logarithm, and it were required to find the natural Number answering to it.

Here the Number sought must consist of Six Places of Figures in whole Numbers, as appears by its Index 5. Which being omitted, I seek in the Table of Logarithms for 660279 but not finding it exactly there, I take the next Less to it, *viz.* 660201, which stands under 3, and against 457. Therefore I conclude, that the first Four Figures of the Number sought, must be 4573; and the Common Difference found against 660201 under *Diff.* is 95.

Then from the given Logarithm 660279

I Subtract the next Less, *viz.* 660201

And there remains 78

To which Annexing Two Cyphers, (*because there is yet wanting two Places of Figures*) it will become 7800: The which being Divided by the Common Difference 95 the Quotient will be 82. Thus 95) 7800 (82 which must be annexed to 4573 (*found before*) and the Sum will be 457382, the Number that answers to the given Logarithm 5.660279 as was requir'd

Thus one may, without much Trouble, find the Logarithm of any given Number (*very near*) altho' it exceed the Limits of the Table by 1, 2, or 3 Places of Figures; And also the Number Agreeing to any given Logarithm, without the help of such a Table of *Proportional Parts* as is usually inserted along with the Table of Logarithms for that Purpose.

Sect. 4. To perform Multiplication by Logarithms.

The Multiplication of any Two given Numbers together, may be perform'd by Logarithms,

	Again, Numbers	Logarithms.	
<i>Dividend</i>	56,73	1.753813	} Sub.
<i>Divisor</i>	0,75	9.875061	
<i>Quotient</i>	79,64	1.878752	

SECT. 6. To Extract the Square and Cube Roots, &c. by Logarithms.

If the Logarithm of any given Number be Divided by 2, the Quotient will be the Logarithm of the *Square Root* of that Number. And if the Logarithm of any Number be Divided by 3, the Quotient will be the Logarithm of the *Cube Root* of that Number: And so on for higher Powers.

Examples.

Suppose it were required to Extract the *Square Root* of 6968, whose Logarithm is 3.843108

Then $2) 3.843108 (1.921554$ This Quotient is the Logarithm of 83,4745 &c. which is the *Square Root* of 6968, as was required.

Again; Let it be required to Extract the *Cube Root* of 6968, whose Logarithm is 3.843108 as before.

Then $3) 3.843108 (1.281036$ This Quotient is the Logarithm of 19,1 &c. which is the *Cube Root* of 6968, As was required.

And in the same manner the *Biquadrat Root* or that of the Fourth Power, may be found, if the Logarithm of the given Number be Divided by 4. And if the Logarithm of any given Number be Divided by 5, the Quotient will be the Logarithm of the *Surfsolid Root* or that of the 5th Power, and so on for the 6th, 7th, 8th, or any proposed Root of a single Power, how high soever it be, provided the Index of the Logarithm of the given Number be Affirmative. But,

The

The Dividing of Logarithms, whose Indices are Negative, (by 2 . 3 . 4 . 5 . &c.) and to determine the true Quotient-Index hath been thought so difficult a Work, that the *Learned* and *Ingenious* Mr. *Oughtred* contrived a Table on purpose to perform it; *Vide Key of the Mathematicks*, pag. 168.

Now according to the Negative Indices I have here made use of, that Work may be very easily perform'd.

Thus { If you are to Divide a Logarithm that hath a Negative Index by 2, Add $\overline{10}$ to the given Index: If by 3, then Add $\overline{20}$ to the given Index: If by 4, then Add $\overline{30}$ to the given Index, &c.

That is, Still increasing the Index of the given Logarithm with $\overline{10}$, as the Divisor doth increase by a Unit or 1.

Examples.

Suppose it were required to Extract the *Square Root* of 0,054756 whose Logarithm is $\overline{8}.738432$

Then $2) \overline{18}.738432$ ($\overline{9}.369216$ This Quotient is the Logarithm of 0,234 which is the *Square Root* of 0,054756, as was requir'd.

Again, Let it be requir'd to Extract the *Cube Root* of 0,054756 whose Logarithm is $\overline{8}.738432$ as before.

Then $3) \overline{28}.738432$ ($\overline{9}.579477$ This is the Logarithm of 0,3797 &c. which is the *Cube Root* of 0,054756 as was requir'd.

Once more; Suppose it were requir'd to Extract the *Second Surfolid Root* or that of the *Seventh Power*, out of 0,054756 whose Logarithm is $\overline{8}.738432$ as above.

Then $7) \overline{68}.738432$ ($\overline{9}.819776$ This is the Logarithm of 0,66035 &c. which is the *Root* requir'd, &c.

Thus

Thus far may suffice concerning the Nature and Use of the Table of Logarithms in general; which being a little consider'd of, it will be easie to Apply them to the following *Calculations*.

C H A P. III.

The **Calculation** of **Questions** in **Simple Interest** and **Annuities**; perform'd both by the *Pen*; and by *Logarithms*:

BEfore we proceed to the following Computations, it may be convenient to premise a few useful things, that will help to Explain and Shorten the Work. And first, Of a few common *Characters*; *Viz.*

- + } Signifies *More*, or the Affirmative Sign of *Addition*.
- } Signifies *Less*, or the Negative Sign of *Subtraction*.
- x } Signifies *Into* or *With*, and is the Sign of *Multiplisatiō*.
- = } Signifies *Equal* to, or the Sign of *Equality*.
- Log. } Denotes the Compleated *Logarithm* of any Number:

And when the *Ratio* of the *Rate of Interest* is mention'd, it signifies only the Simple Interest of 1*l.* for one Year; at any proposed Rate of Interest *per Cent.* which may be thus found by the *Rule of Three*.

As 100 : Is to 6 :: So is 1 : To 0,06 the *Ratio* of the Rate of 6 *per Cent. per Annum*.

Or, As 100 : Is to 7 :: So is 1 : To 0,07 the *Ratio* of the Rate of 7 *per Cent. per Annum, &c.*

The which may also be found by Logarithms.

Thus,

Thus, { From the Logarithm of the given Rate of Interest, Subtract the Logarithm of 100 (viz. 2.000000) and the Remainder will be the Logarithm of the Ratio of that Rate.

Example.

Suppose the given Rate of Interest to be that of 6 per Cent. per Annum : Then

The Logarithm of 6 is 0.778151 } Subtract
The Logarithm of 100 is 2.000000 }

The Remainder is the Log. 8.778151 of 0,06

That is, 0,06 is the Ratio of 6 per Cent. &c.

And thus may the Ratio of any other proposed Rate of Interest per Cent. be easily obtain'd.

But because 'tis the Logarithms of those Ratio's, that are of use in the following Calculations relating to Simple Interest, I have here annexed a small Table of several Rates of Interest, with their Ratio's, and the Logarithms of those Ratio's.

Rates of Interest per Cent.	Ratio's of those Rates.	Logarithms of those Ratio's.	Rates of Interest per Cent.	Ratio's of those Rates.	Logarithms of those Ratio's.
3	0,03	8.477121	7	0,07	8.845098
4	0,04	8.602060	7½	0,075	8.875061
4½	0,045	8.653212	8	0,08	8.903090
5	0,05	8.698970	8½	0,085	8.929419
5½	0,055	8.740363	9	0,09	8.954242
6	0,06	8.778151	9½	0,095	8.977724
6½	0,065	8.812913	10	0,1	9.000000

These things being premised, we may proceed to the following Work ; and first of Money forborn at any Rate of Simple Interest, &c.

SECT. I.

Sect. I. Of Simple Interest.

In the *Young Mathematician's Guide*, pag. 245. (from whence the following Rules are deduc'd) I've made use of Letters to denote the several Parts of the Question,

Vi2. { *P.* Signifies any Principal or Sum put to Interest.
T. The Time of its continuing at Interest.
R. The Ratio of the Rate of Interest *per Cent.*
A. The Amount of the Principal and its Interest.

Any three of these Parts being given, the other may be found by help of this General

$$\textit{Theorem, } TRP + P = A.$$

This Theorem admits of four Cases or Variety of Questions.

Case 1. If *P*, *T*, and *R*, are given, thence to find *A*. That is, If any Principal, with the Time of its being at Interest, and the Rate of Interest *per Cent. per Annum* are given, To find the Interest, and the Amount.

This Question I take to be of the most general Use of any that Occurs in the whole Business of Simple Interest; and may be perform'd thus:

First by Common *Arithmetick*.

Rule. { Multiply the Principal, the Time, and the Ratio of the Rate, all three together; and their Product will be the Interest: To which Add the Principal, and the Sum will be the Amount requir'd.

Example.

What Sum will 567l. 10s. Amount to in Nine Years, at the Rate of 6 per Cent. per Annum?

Here is given $P = 567,5$ $T = 9$ and $R = 0,06$ To find *A*; which by the Rule is done thus:

First

First $P = 567,5$ } *Multiply.* Again, $5107,5$ } *Multiply.*
 And $T = 9$ } $R = 0,06$ }

Product $5107,5$ $306,45 = 306\text{ l. } 9\text{ s.}$

That is, $306\text{ l. } 9\text{ s.}$ is the Interest of $567\text{ l. } 10\text{ s.}$ for Nine Years : Then $306\text{ l. } 9\text{ s.} + 567\text{ l. } 10\text{ s.} = 873\text{ l. } 19\text{ s.} = A$, the Amount requir'd.

The same perform'd by *Logarithms*.

Thus, { *To the Log. of the Principal, Add the Log. of the Time, and the Log. of the Ratio ; their Sum will be the Log. of the Interest ; to which Add the Principal, &c. As above.*

That is, in the same *Example*.

Thus, $P = 567,5$ its Log. is 2.753966 }
 And, $T = 9$ its Log. is 0.954242 } *Add*
 $R = 0,06$ its Log. is 8.778151 }

The Sum is the Logarithm 2.486359 of $306,45$

That is, $306\text{ l. } 9\text{ s.}$ is the Interest as before, to which the Princip. $567\text{ l. } 10\text{ s.}$ being Added,

the Sum is $873\text{ l. } 19\text{ s.} = A$, the Amount requir'd, as above.

Case 2. When A , T , and R , are given ; To find P :
 That is, To find what Principal or Sum, being put to Interest any assigned Time, will Amount to a proposed Sum in that Time, at any given Rate of Interest per Cent. per Annum, &c.

First by the *Pen* only.

Rule. { *Multiply the Time with the Ratio of the Rate, and to their Product Add 1 : Then Divide the proposed Amount by that Sum, and the Quotient will shew the Principal required.*

Example.

What Principal or Sum of Money, being put to Interest for Nine Years, will Amount to (or Raise a Stock of)
 $873\text{ l. } 19\text{ s.}$ at 6 per Cent. &c. ? Or

Or thus ; Suppose a Debt of 873 l. 19 s. w^{ers} not to be paid until Nine Years hence ; What would it be worth in ready Money ; the Creditor allowing the Rate of 6 per Cent. Discount to the Debtor ?

In this Question there is given, $A = 873,95$ $T = 9$ and $R = 0,06$ To find P ,

Thus $T = 9$ } $R = 0,06$ } *Mult.* Then $0,54 + 1 = 1,54$
 And $1,54 \times 873,95 = 567,5 = P$.
Product $0,54$

That is, 567,5 = 567 l. 10 s. is the Principal, (or Ready Money) requir'd.

The same may be perform'd by Logarithms,

Thus, { Add the Log. of the Time to the Log. of the Ratio of the Rate, and to the Number found by their Sum Add 1. Then if the Log. of that Sum be Subtracted from the Log. of the proposed Amount, there will Remain the Log. of the Principal requir'd.

That is, in the same Example,

Thus $T = 9$ its Log. is $0,954242$ }
 And $R = 0,06$ its Log. is $8,778151$ } *Add*

The Sum is this Logar. $9,732393$ of $0,45$ To which Adding 1, it will become $1,45$.

Then $A = 873,95$ its Log. is $2,941487$ }
 And $1,54$ its Log. is $0,187521$ } *Subtract*

Remains the Logarithm $2,753966$ of $567,5 = P$
 That is, 567 l. 10 s. is the Principal required by the Question, as above.

Case 3. Suppose P , T , and A , were given ; To find R , viz. Having any Principal with the Time of its being at Interest, and the Sum it's proposed to Raise or Amount to in that Time, given ; thence to find the Rate of Interest per Cent. per Annum.

First

First by the Pen only.

Rule. { If the Difference between the proposed Amount and the Principal, be Divided by the Product of the Principal Multiplied into the Time, the Quotient will shew the Ratio of the Rate of Interest requir'd.

Example.

At what Rate of Interest per Cent. will 567 l. 10 s. Raise a Stock, or Amount to 873 l. 19 s. in Nine Tears time?

Here is given $P = 567,5$ $T = 9$, and $A = 873,95$
To find R ; which by the Rule is

Thus, $873,95 - 567,5 = 306,45$ the Dividend.

And $567,5 \times 9 = 5107,5$ the Divisor.

Then $5107,5 \div 306,45 (0,06 = R$, the Ratio of the Rate of Interest;

And As 1 : Is to 0,06 :: So is 100 : To 6 the Rate of Interest per Cent. &c. as was required.

Or by Logarithms,

Thus, { From the proposed Amount Subtract the Principal (as above;) then from the Log. of the Remainder, Subtract the Sum of the Logarithms of the Principal and the Time, and there will Remain the Log. of the Ratio of the Rate, &c.

That is, in the same Example,

$873,95 - 567,5 = 306,45$ its Log. 2.486359

And $P = 567,5$ its Logarith. is 2.753966 } Add
 $T = 9$ its Logarith. is 0.954242 }

From the first Log. Subst. this Log. 3.708208

And there remains the Logarith. 8.778151 of 0,06

Which shews the Rate of Interest to be 6 per Cent. per Annum, as before by the Pen.

Case 4. Having P , R , and A , given, To find T :
That is, To find the Time in which any given Principal will Amount to, or Raise any proposed Sum, the Rate of Interest per Cent. being also given.

First by the *Pen* only.

Rule. $\left\{ \begin{array}{l} \text{If the Difference between the proposed} \\ \text{Amount and the Principal, be Divided by the} \\ \text{Product of the Principal Multiplied into the} \\ \text{Ratio of the Rate; the Quotient will shew the} \\ \text{Time requir'd.} \end{array} \right.$

Example.

In what Time will 567 l. 10 s. Amount to (or Raise a Stock of) 873 l. 19 s. at 6 per Cent. per Annum?

In this Quest. there is given $P = 567,5$ $R = 0,06$ and $A = 873,95$ To find T ; which is done thus:

First $873,95 - 567,5 = 306,45$ the Dividend.

And $567,5 \times 0,06 = 34,05$ for the Divisor.

Then $34,05 \overline{) 306,45}$ ($9 = T$ the Time; *Viz.* Nine Years will be the Time required.

The same perform'd by *Logarithms*.

Thus, $\left\{ \begin{array}{l} \text{From the proposed Amount Subtract the Prin-} \\ \text{cipal (as before;)} \text{ Then from the Logarithm} \\ \text{of the Remainder, Subtract the Sum of the} \\ \text{Logarithms of the Principal and Ratio of the} \\ \text{Rate; and there will Remain the Logarithm of} \\ \text{the Time.} \end{array} \right.$

That is, in the Last Example;

Thus $873,95 - 567,5 = 306,45$ its Log. 2.486359

$P = 567,5$ its Logarith. is 2.753966 } Add
 $R = 0,06$ its Logarith. is 8.778151 }

From the first Log. Subst. this Log. 1.532117

There remains the Logar. of $9 = T$ 0.954242

Which shews, that the Time sought is just Nine Years.

If the Work of these Four Examples, and the Rules by which they are perform'd, be well understood, they will be sufficient (*notwithstanding there is really but*

but One Question, only it is varied according to the several Cases) to shew how any Question of the like kind may be truly Resolv'd, at any proposed Rate of Simple Interest, and for any assign'd Time; especially if the Time given (or sought) does consist of compleat or whole Years.

But if the Time given (or sought) does not consist of whole Years, as most generally it does not, it being either Less than a Year; or Years, and some Parts of a Year, as *Weeks, Months, or Quarters &c.* Then the odd Time, less than a compleat Year, must be Reduced (or Converted) into Decimal Parts of a Year (*per Sect. 5. p. 7.*) And unless such Parts of a Year chance to be just $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ of a Year, then the best way will be to Reduce the Odd Time into Days, and then work with the Decimal Parts of a Year that are equivalent to those Number of Days.

And for the ease and ready finding out of the true Number of Days that are contain'd between any two assigned Times less than a Year, and the Decimal Parts of a Year that are equal to those Days, I have here inserted two small Tables.

The Use of the following Table of Months and Days, is no more but thus; Find the first proposed Month at the Top or Head of its respective Part of the Table; and in the same Column under it, look for the other Month, and by it stands the Number of Days requir'd, according to the Title of the Table.

As for Example; From the 1st, 5th, 10th, 17th, or any other Day in *April*, To the 1st, 5th, 10th, 17th (*viz. to the same*) Day in *December*, is just 244 Days.

Or, From the 4th, 7th, or 12th, &c. of *October*, To the 4th, 7th, or 12th, &c. of *August* is just 304 Days. And so the true Number of Days that are between any two of the same Days in the proposed Months, may be found by Inspection only.

But if the two given Days of the Month are different, then their Difference must be Added to the Number found in the Table: As, suppose, between the

4th of *October*, and the 25th of *August*; Here, because the 25th of the one Month, is 21 Days more than the 4th Day of the other Month; therefore the Number of Days requir'd will be $304 + 21$; viz. 325 Days, and so for any other two Months and Number of Days in any proposed Time throughout the Year; as in this Table.

A Table that shews, by Inspection only, the true Number of Days, from every Day in any Month, to the same Day in any other Month, throughout the whole Year.

January	Febru.	March	April	May	June
Feb. 31	Mar. 28	Apr. 31	May 30	June 31	July 30
Mar. 59	Apr. 59	May 61	June 61	July 61	Aug. 61
Apr. 90	May 89	June 92	July 91	Aug. 92	Sept. 92
May 120	June 120	July 122	Aug. 122	Sept. 123	Oct. 122
June 151	July 150	Aug. 153	Sept. 153	Oct. 153	Nov. 153
July 181	Aug. 181	Sept. 184	Oct. 183	Nov. 184	Dec. 183
Aug. 212	Sept. 212	Oct. 214	Nov. 214	Dec. 214	Jan. 214
Sept. 243	Oct. 242	Nov. 245	Dec. 244	Jan. 245	Feb. 245
Oct. 273	Nov. 273	Dec. 275	Jan. 275	Feb. 276	Mar. 273
Nov. 304	Dec. 303	Jan. 306	Feb. 306	Mar. 304	Apr. 304
Dec. 334	Jan. 334	Feb. 337	Mar. 334	Apr. 335	May 334
Jan. 365	Feb. 365	Mar. 365	Apr. 365	May 365	June 365

July	August	Septem.	Octob.	Novem.	Decem.
Aug. 31	Sept. 31	Oct. 30	Nov. 31	Dec. 30	Jan. 31
Sept. 62	Oct. 61	Nov. 61	Dec. 61	Jan. 61	Feb. 62
Oct. 92	Nov. 92	Dec. 91	Jan. 92	Feb. 92	Mar. 90
Nov. 123	Dec. 122	Jan. 122	Feb. 123	Mar. 120	Apr. 121
Dec. 153	Jan. 153	Feb. 153	Mar. 151	Apr. 151	May 151
Jan. 184	Feb. 184	Mar. 181	Apr. 182	May 181	June 182
Feb. 215	Mar. 212	Apr. 212	May 212	June 212	July 212
Mar. 243	Apr. 243	May 242	June 243	July 242	Aug. 243
Apr. 274	May 273	June 273	July 273	Aug. 273	Sept. 274
May 304	June 304	July 303	Aug. 304	Sept. 304	Oct. 304
June 335	July 334	Aug. 334	Sept. 335	Oct. 334	Nov. 335
July 365	Aug. 365	Sept. 365	Oct. 365	Nov. 365	Dec. 365

A Table for the ready finding the Decimal Parts of a Year, Equal to any Number of Days, &c.

<i>Days. Dec. parts</i>	<i>Days. Dec. parts</i>	<i>Days. Dec. parts</i>
1 = 0,002740	10 = 0,027397	100 = 0,273973
2 = 0,005479	20 = 0,054794	200 = 0,547945
3 = 0,008219	30 = 0,082192	300 = 0,821918
4 = 0,010959	40 = 0,109589	365 = 1,000000
5 = 0,013699	50 = 0,136986	$\frac{1}{4}$ of a Year = 0,25
6 = 0,016438	60 = 0,164383	$\frac{1}{2}$ a Year = 0,5
7 = 0,019178	70 = 0,191781	$\frac{3}{4}$ of a Year = 0,75
8 = 0,021918	80 = 0,219178	
9 = 0,024657	90 = 0,246575	

The Use of this Table is thus :

If the proposed Number of Days can be exactly found in the Table (under *Days*) their Decimal Parts are also found against them by Inspection only.

But if the true Number of Days cannot be exactly found there, then both they, and their Decimal Parts must be collected out of the Table at twice, or thrice, according as their Number requires.

As for Example : Suppose it were required to find the Decimal Parts of a Year equal to 135 Days ?

Days. Dec. parts.

Then $\left\{ \begin{array}{l} 100 = 0,273973 \\ 30 = 0,082192 \\ 5 = 0,013699 \end{array} \right\}$ Add all these together.

Hence 135 = 0,369864 the Decimal Parts required.

And thus may the Decimal Parts of a Year, equivalent to any given Number of Days, be very easily found to Six Places of Figures; but for Common Business it may suffice to work with only Four of those Places, and in small Sums, 2, or 3 Places (according to Discretion) may be near enough to the Truth

Or the Decimal Parts, equivalent to any given Number of Days, may be found by the Logarithms.

D 3

Thus,

Thus, { From the Log. of the given Number of Days,
Subtract the Log. of 365 Days, (viz. 2.562293)
and there will remain the Log. of the Decimal
parts equal to those Number of Days.

Example.

Let it be required to find the Decimal Parts of a Year equal to 135 Days; As before.

Here is given 135 its Logar. is 2.130334 } Subtract
Days in a Year 365 its Log. is 2.562293 }
And there remains the Log. 9.568041 of 0.369863;
the Decimal Parts equal to 135 Days, &c. as above.

These things being understood, it will be as easy to Calculate any Question relating to any of the foregoing Cases, when the Time given, or sought is either Less than a Year, or Years and Parts of a Year; as it is for those in whole Years only.

As for Instance in Case 1. Suppose it were required to find *What Sum 780 l. 15 s. would Amount to in 3 Years and 179 Days, at Five and a Half per Cent.?*

In this Quest. there is given $P = 780.75$ $R = 0.055$ and $T = 3.4904$ found by the Table.

Thus, Three Years = 3.000000 }
Days { 100 = 0.273973 } Add
70 = 0.191781 }
9 = 0.024657 }

Hence 3 Years and 179 Days is = 3.490411 = T .

Then $P = 780.75$ its Log. 2.892512 }
 $T = 3.4904$ its Log. 0.542875 } Add
 $R = 0.055$ its Log. 8.740363 }

The Sum is the Logarithm 2.175750 of 149.88
That is, 149.88 = 149 l. 17 s. 7 d. the Inter. To which
Add the Principal 780 l. 15 s. 0 d.

And the Sum is 930 l. 12 s. 7 d. the Amount requir'd.

Again,

Again, That I may make all as plain as I can, take an *Example* in *Case 4. viz.* To find the Time.

Let it be requir'd to find, *In what Time* 780 l. 15 s. would Amount to the Sum of 930 l. 12 s. 7 d. at the Rate of $5\frac{1}{2}$ per Cent. per Annum?

In this Example there is given $A = 930,63$ $R = 0,055$ and $P = 780,75$ To find T : Which is done thus:

$$930,63 - 780,75 = 149,88 \text{ its Log. } 2.175750$$

$$\text{And } P = 780,75 \text{ its Log. } 2.892512 \quad \left. \begin{array}{l} \\ R = 0,055 \text{ its Log. } 8.740363 \end{array} \right\} \text{Add}$$

From the first Log. Subst. this Log. 1.632875

And there remains the Log. 0.542875 of 3.4904.

That is, 3 Years, and 0,4904 Decimal Parts of a Year; but $0,4904 = 179$ Days, per Rule in *pag. 8.* Consequently 3 Years and 179 Days is the Time requir'd by the Question.

Or the Decimal Parts of a Year may be otherwise Reduced into Days, by help of the last Table;

As in this Example; From 0,4904 the given Parts, Take the next less Tabular N°. $0,2740 = 100$ Days.

There remains $0,2164$

Again, From the Rem. take $0,1918 = 70$ Days.

Lastly this $0,0246 = 9$ Days.

So that $0,4904 = 179$ Days, by the Table, &c. as before.

Or the Decimal Parts of a Year may be Reduced into Days by the Logarithms;

Thus, $\left\{ \begin{array}{l} \text{To the Log. of the proposed Parts, Add the Log.} \\ \text{of 365, the Days in a Common Year, and the} \\ \text{the Sum will be the Log. of the Days requir'd.} \end{array} \right.$

For Instance, In the Last Example, wherein the given Decimal Parts are 0,4904 Log. 9.690550 } Add
The Days in a Year are 365 its Log. 2.562293

The Sum is this Logarithm 2.252843 of 179
Days.

**Sect. 2. Of Annuities, or Pensions, &c.
in Arrears, Computed at Simple Interest.**

I shall here make use of the same Letters to denote the several Parts of the Question, as in the aforesaid *Young Mathematician's Guide*, p. 248.

Viz.

- U. { Denotes the Annuity or Pension, &c. *viz.* either
Yearly, Half Yearly, or Quarterly Rents.
T. { The Time of its being unpaid; *viz.* the Number
of all the Payments that are in Arrears.
A. { The Amount of the Annuity and its Interest;
viz. the Sum of all the Arrears due,
R. { The Ratio of the Rate, *viz.* the Interest of 1 l.
As before.

$$\text{Then will } \left\{ \frac{TTR - TR}{2} = \frac{A - TU}{U} \right.$$

From this Equation is deduced the following Rules, which admits of Four Cases.

Note. I do here take it for granted, that the Reader doth by this time very well know how to perform Multiplication, and Division of any given Numbers, by Logarithms; (as directed in Chap. II. and practised in all the Examples of the Last Section :) And therefore I shall, for Brevity sake, omit setting down in Words at length (as in the last Section) how the following Cases are to be Resolv'd by Logarithms; that Work being so easily understood, by having a due regard to their respective Rules; That, it seems to me, to be wholly needless to repeat in Words how it is to be done; But only to set down the Work, at large of all the Examples, according to the Import of their respective Rules, by which they are Computed with the Pen only.

Case

Case 1. Having U , T , and R , given; To find A . That is, If any Annuity, or Rent, with the Time of its being Unpaid, and the Rate of Interest *per Cent.* be given; thence to find what Sum all these Arrears will Amount to in that Time; allowing any assigned Rate of Simple Interest, for every particular Payment as it becomes due.

Rule. $\left\{ \begin{array}{l} \text{Multiply the Time, the Time less 1, and Half} \\ \text{the Ratio of the Rate all three together, and} \\ \text{to their Product Add the Time; Then Multiply} \\ \text{that Sum with the Annuity, and the Product} \\ \text{will shew the Amount requir'd.} \end{array} \right.$

Example.

Suppose 356 l. Annuity, or Yearly Rent, be forborn or unpaid Nine Years; What Sum will all those Arrears Amount to in that Time, allowing 6 per Cent. per Ann. for each Payment as it becomes due?

In this Example there is given, $U = 356$; $T = 9$; and $R = 0,06$; to find A .

First by the Pen only.

Let the Work be prepared thus, $9 - 1 = 8$,
And $\frac{1}{2} R = 0,03$

Then $9 \times 8 = 72$; and $72 \times 0,03 = 2,16$; To which Adding the Time, it will be $2,16 + 9 = 11,16$. And 11,16 Multiplied with 356 is $3972,96 = A$.

That is 3972 l. 19 s. 2½ d. will be the Amount as was requir'd.

The same perform'd by *Logarithms*.

First $T = 9$ its Logar. is 0.954242
And $T - 1 = 8$ its Logar. is 0.903090
Again, $\frac{1}{2} R = 0,03$ its Log. is 8.477121 } Add

Their Sum is the Log. 0.334453 of 2.16

Then $2,16 + 9 = 11,16$ its Log. is 1.047664 } Add
And $U = 356$ its Log. is 2.551450

The Sum is the Logarithm 3.599114 of 3972,96

That is 3972 l. 19 s. 2½ d. = A , the Amount or Sum of all the Arrears, as before by the Pen.

Case

Case 2. When A , T , and R , are given, to find U . That is, To find what Annuity, or Yearly Rent, being forborn or unpaid any assign'd Time, will Amount to a proposed Sum; allowing any given Rate of Interest per Cent. for every Payment as it becomes due.

Rule. $\left\{ \begin{array}{l} \text{Multiply the Time, the Time less 1, and Half} \\ \text{the Ratio of the Rate, all three together; and to} \\ \text{their Product, Add the Time (as before) Then} \\ \text{Divide the proposed Amount by that Sum, and} \\ \text{the Quotient will shew the Annuity, or Yearly} \\ \text{Rent requir'd.} \end{array} \right.$

Example.

Suppose it were requir'd to find, What Annuity, or Yearly Rent will Amount to (or Raise a Stock) of 3972 l. 19 s. 2½ d. allowing 6 per Cent. for every Payment as it becomes due?

In this Question there is given $A = 3972,96$; $T = 9$; and $R = 0,06$ To find U ; which may be thus done.

First by the Pen only; Thus,

The Work prepared is $9 - 1 = 8$, and $\frac{1}{2} R = 0,03$ Then $9 \times 8 = 72$, and $72 \times 0,03 = 2,16$; which being Added to the Time, is $9 + 2,16 = 11,16$ for a Divisor.

Then $11,16 \overline{) 3972,96} \{ 356 = U$
Viz. 356 l. will be the Yearly Rent requir'd.

The same perform'd by Logarithms.

Thus, $T = 9$ its Log. is 0.954242
 $T - 1 = 8$ its Log. is 0.903090
 And $\frac{1}{2} R = 0,03$ its Log. is 8.477121 $\left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Add}$

The Sum is the Logarithm 0.334453 of 2,16

Again, $A = 3972,96$ its Log. is 3.599113
 And $9 + 2,16 = 11,16$ its Log. 1.047664 $\left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Subtract}$

There remains the Logarithm 2.551449 of 356 = U
 That is, 356 l. is the Annuity, or Yearly Rent, As before.

Case

Case 3. When U , A , and R , are given; To find T .
That is, To find the *Time* in which any Annuity,
or Yearly Rent, being forborn or unpaid, will Amount
to any proposed Sum; allowing any given Rate of In-
terest *per Cent.* &c. for each Payment as it becomes due.

Rule. { *Subtract the Ratio of the Rate from 2; then*
Divide the Remainder by twice the Ratio, and
call the Quotient x .
Next, Square that Quotient (viz. Multiply it
with it self) and call that Square xx .
Then Divide Twice the proposed Amount by
the Product of the Annuity Multiply'd with the
Ratio, and to the Quotient, Add the Square
Number called xx ; Then Extract the Square
Root of that Sum, and from that Root Sub-
tract the Number called x , and the Remainder
will shew the Time sought.

Example.

What Time will 356 l. Yearly Rent require to Raise
the Sum of 3972 l. 19 s. $2\frac{1}{2}$ d. at Six per Cent. &c.
for each Payment as it becomes due?

In this Question there is given, $U = 356$; $R = 0,06$;
and $A = 3972,96$; To find T .

And first by the *Pen* only.

1st $2 - 0,06 = 1,94$ And $2 R = 0,12$) $1,94$ ($16,166 = x$

Next, $16,166 \times 16,166 = 261,361$ the Num. call'd xx

Then $2 A = 7945,92$ and $356 \times 0,06 = 21,36$
And $21,36$) $7945,92$ ($372,003$

Then $372,003 + 261,361 = 633,364$ whose Square
Root is $25,166$; Lastly, $25,166 - 16,166$ Leaves
 $9 = T$; *Viz.* 9 Years, is the Time requir'd by the
Question.

The

The same found by *Logarithms*.

First $2 - 0,06 = 1,94$ its Log. 0.287802 } Subtract
 And $2R = 0,12$ its Log. is 9.079181 }

There Remains the Logar. 1.208621 of $16,166$
 Multiplied with 2 (call'd x .)

The Product is the Logarithm 2.417242 of $261,361$
 (call'd xx .)

Again; $2A = 7945,92$ its Log. 3.900145

And $U = 356$ its Log. is 2.551449 } Add
 $R = 0,06$ its Log. is 8.778151 }

From the 1st Log. Sub. this Log. 1.329600

And there remains the Logar. 2.570545 of $372,003$

Then $372,003 + 261,361 = 633,264$ its Log. 2.801585
 The Half of that Log. is 1.400792

Lastly, the Number which belongs to the Half Logarithm is $25,166$ From which Subst. the Number called $x = 16,166$

Remains $9,000 = T$; the Num. of Years sought, &c.

Case 4. If U , T , and A , are given; To find R .
 That is, Having any Annuity, or Yearly Rent, with the Time of its being Unpaid, and the Sum it's proposed to Amount to in that Time given; Thence to find, what Rate of Interest *per Cent.* must be allow'd for every Payment as it becomes due.

Rule. $\left\{ \begin{array}{l} \text{Multiply the Annuity with the Time, and} \\ \text{Multiply that Product with the Time again;} \\ \text{And make half the Difference of those Two} \\ \text{Products a Divisor.} \end{array} \right.$
 $\left\{ \begin{array}{l} \text{Next, Subtract the first Product from the} \\ \text{proposed Amount, then Divide the Remainder} \\ \text{by the aforesaid Divisor, and the Quotient will} \\ \text{be the Ratio of the Rate of Interest required.} \end{array} \right.$

Exam-

Example.

Suppose 356 l. Yearly Rent, being forborn or unpaid Nine Years, be proposed to Raise the Sum of 3972 l. 19 s. 2 $\frac{1}{4}$ d. What Rate of Interest per Cent. must be allow'd for every Payment as it becomes due ?

In this Question there is given $U=356$, $T=9$; And $A=3972,96$ To find R . Which may be thus found.

First by the Pen only.

Thus, $356 \times 9 = 3204$; And $3204 \times 9 = 28836$

Next, $28836 - 3204 = 25632$ its Half is 12816 for the Divisor.

And $3972,96 - 3204 = 768,96$ for the Dividend. Then $12816 \mid 768,96$ (0,06 = R the Ratio.

And it will be, As 1 : Is to 0,06 :: So is 100 : To 6 the Rate of Interest per Cent. &c. As was required.

The same may be done by Logarithms.

Thus $U=356$ its Log. 2.551450 } Add
And $T=9$ its Log. 0.954242 }

To the last Log. Add this 3.505692 its Num. is 3204

Their Sum 4.459934 its Num. is 28836

The Difference of the Two Numbers is 25632 and the $\frac{1}{2}$ of 25632 is 12816 for the Divisor.

From $A=3972,96$ take the 1st Num. 3204, and there remains 768,96 its Log. 2.885904 } Subtract
and 12816 the Divis. its Log. 4.107752 }

There Remains the Logar. 8.778152 of 0,06 = R
Then 1 : 0,06 :: 100 : 6 the Rate of Interest per Cent. &c.

Thus you have all the Four Cases relating to Annuities, or Rents, &c. in Arrears, with their Examples
E in

in Yearly Payments : But if the *Annuities*, or *Rents*, are to be paid by Half-yearly, or Quarterly Payments, as most generally they are,

Then,

1. Instead of the Ratio of the given Rate of Interest, you must take the $\frac{1}{2}$ of that Ratio for Half-yearly Payments, and the $\frac{1}{4}$ of it for Quarterly Payments, &c.

2. And you must take the $\frac{1}{2}$ of the Yearly Rent for Half-yearly Payments, and the $\frac{1}{4}$ of it for Quarterly Payments, &c.

3. But instead of the proposed Number of Years, you must take Twice that Number for Half-yearly Payments, and Four times that Number for Quarterly Payments, &c. As in the following Examples.

Examples in Half-yearly Payments.

Suppose 356 l. per Annum Annuity, payable every Half-year, were forborn or unpaid Nine Years ; What would all those Arrears Amount to, at the Rate of 6 per Cent. per Annum, &c. ?

In this Question there is given $U = 178$, viz. the $\frac{1}{2}$ of 356 l. $R = 0,03$ the $\frac{1}{2}$ of the Ratio of 6 per Cent. and $T = 18$, viz. 9×2 the Number of Half-years in Nine Years ; Thence to find A . (per Rule at Case 1.)

First to prepare the Work $18 - 1 = 17$;
and $\frac{1}{2} R = 0,015$.

Then $18 \times 17 = 306$; and $306 \times 0,015 = 4,59$
To which Add the Time, viz. $4,59 + 18 = 22,59$
And then $22,59 \times 178$ gives $4021,02 = A$, the Amount or Sum of all the Arrears, as was requir'd.

The same Example in Quarterly Payments.

That is, 356 l. a Year, to be paid every Quarter, being forborn Nine Years, What Sum will it Amount to, allowing 6 per Cent. &c. for every Payment as it becomes due ?

Now

Now here it will be $U = 89$, viz. the $\frac{1}{4}$ of 356 l.
 $R = 0,015$, viz. the $\frac{1}{4}$ of 0,06 the Ratio of the gi-
 ven Rate per Cent. &c. And $T = 36$, viz. 9×4 the
 Number of Quarters in Nine Years: Thence to find
 A , as before.

To Prepare the the Work $36 - 1 = 35$,
 and $\frac{1}{2} R = 0,0075$.

Then $36 \times 35 = 1260$; and $1260 \times 0,0075 = 9,45$
 to which Add the Time, viz. $9,45 + 36 = 45,45$,
 and then $45,45 \times 89$ gives $4045,05 = A$, the
 Amount or Sum of all the Arrears at Quarterly
 Payments.

By comparing the Work of these Two Examples
 with that in Page 33, it may be observ'd, That
 Half-yearly Payments are more advantagious than
 Yearly Payments; and Quarterly are yet more ad-
 vantagious than Half-yearly:

For Yearly Payments Amount but to 3972 l. 19 s. 2 $\frac{1}{2}$ d.
 But $\frac{1}{2}$ yearly Payments Amounts to 4021 l. 00 s. 3 $\frac{1}{2}$ d.
 And Quarterly Paym. Amounts to 4045 l. 01 s. 00 d.

If the two last Examples be well consider'd and un-
 derstood, it will be easie to conceive how to Com-
 pute any Question in the other Three Cases, when
 the Payments are either Half-yearly, or Quarterly;
 and therefore I shall omit inserting Examples, and
 proceed to the next Section.

SECT. 3. The Present Worth of Annuities and Pensions, &c. Computed at Simple Interest.

In this Section I shall make use of these Letters to
 denote the several Parts of the Question:

Viz. $\left. \begin{array}{l} U. \text{ Denotes the Annuity or Rent.} \\ T. \text{ The Time of its Continuance.} \\ R. \text{ The Ratio of the Rate of Interest.} \end{array} \right\} \text{As before.}$

And P . Denotes the Present Worth of the Annuity.

$$\text{Then } TTRU - TRU + 2TU = 2P + 2TRP$$

(Vide the afore said *Young Mathematician's Guide*, p. 251.)

And from this Equation are deduced the following Rules, which also admits of Four Cases, as in the last Section.

Case 1. Having *U*, *T*, and *R*, given; To find *P*.

That is, Having any Annuity or Yearly Rent, and the Time of its Continuance given; To find the Present Worth of that Annuity, or Lease, &c. allowing any given Rate of Simple Interest per Cent. to the Purchaser for his ready Money.

Rule. $\left\{ \begin{array}{l} \text{Multiply the Time, the Time less 1, and Half} \\ \text{the Ratio of the Rate, all three together, and to} \\ \text{their Product Add the Time; Then Multiply that} \\ \text{Sum with the Annuity. (So far the Work is} \\ \text{just the same as in the last Section.) Then Di-} \\ \text{vide that Product, by the Product of the Ratio} \\ \text{into the Time, Added to 1. and the Quotient} \\ \text{will shew the Present Worth requir'd.} \end{array} \right.$

Example.

Suppose a Lease of 250 l. per Annum, the Rents to be paid Half-yearly, were to be Lett for 21 Years; What may the present Worth of that Lease be, at the Rate of 5 per Cent. &c.

In this Question there is given $U = 125$, viz. the $\frac{1}{2}$ of 250; $R = 0,025$ the $\frac{1}{2}$ of 0,05 the Ratio of 5 per Cent.; and $T = 42$, (viz. 21×2) the Number of Half-Years in 21 Years; To find *P*.

First, the Work Prepared stands thus, $42 - 1 = 41$
and $\frac{1}{2} R = 0,0125$.

Then $42 \times 41 = 1722$; and $1722 \times 0,0125 = 21,525$
Next, $21,525 + 42 = 63,525$ which Multiplied with 125, the Half-yearly Payment, is 7940,625 for the Dividend.

Again,

Again, $42 \times 0,025 = 1,05$, and $1,05 + 1 = 2,05$ for the Divisor.

Then $2,05 \} 7940,625 (3873,4756 = P$.
Viz. 3873 l. 9 s. 6 d. will be the Present Worth of such a Lease as was requir'd.

The same perform'd by *Logarithms*.

First $T = 42$ its Log. is 1.623249
 And $T - 1 = 41$ its Log. is 1.612784 } Add
 $\frac{1}{2} R = 0,0125$ its Log. is 8.096910
 The Sum is the Logarith. 1.332943 of 21,525
 And $21,525 + 42 = 63,525$ its Log. 1.802945 } Add
 $U = 125$ its Log. is 2.096910
 The Sum is the Log. of the Dividend 3.899855

Again, $T = 42$ its Logar. is 1.623249 } Add
 And $R = 0,025$ its Log. is 8.397940
 The Sum is the Logarithm 0.021189 of 1,05
 And $1,05 + 1 = 2,05$ for the Divisor.

Then the Log. of the Divid. is 3.899855 } Subtract
 And the Log. of 2,05 the Divif. 0.311754

There Remains the Log. 3.588101 of 3873,47
 That is, 3873 l. 9 s. 4 $\frac{1}{4}$ d. is here found to be the Present Worth of that Annuity; As before, *Ferè*.

Case 2. When P , T , and R , are given; to find U .
Viz. To find what Annuity, or Lease, &c. may be Purchased for any proposed Sum, to continue any assigned Time, and at any given Rate of Interest.

Rules { Multiply the Time, the Ratio of the Rate, and the present Worth, (*viz.* the Purchase-Money) all three together; and to their Product Add the present Worth; make that Sum a Dividend.
 { Then Multiply the Time, the Time less 1, and half the Ratio together; and to their Product Add the Time; make that Sum a Divisor; the Quotient arising from thence, will shew the Annuity, or Rent required. *Exam-*

Example.

What Annuity, or Rent, to be paid Half-yearly, and so continue 21 Years, may be Purchased for the Sum of 3873*l.* 9*s.* 6*d.* at the Rate of 5 per Cent.?

In this Example there is given $P = 3873,475$
 $T = 42$ viz. 21×2 , the Half-Years in 21 Years, and
 $R = 0,25$ the $\frac{1}{4}$ of the Ratio 0,05 of 5 per Cent.; To find U , the Half-yearly Rent.

The Work Prepared is $42 - 1 = 41$; and $\frac{1}{2} R = 0,0125$

Then $3873,475 \times 42 = 162685,95$ and $162685,95 \times 0,025$ gives 4067,14875; And $4067,14875 + 3873,475 = 7940,62375$ for the Dividend.

Again, $42 \times 41 = 1722$ and $1722 \times 0,0125 = 21,525$ and $21,525 + 42 = 63,525$ for the Divisor.

Then $63,525 \mid 7940,62375$ ($125 = U$, the Half-yearly Rent, consequently $125 \times 2 = 250$ *l.* will be the Annuity or Rent *per Annum*, as was required.

The same done by *Logarithms*.

First $P = 3873,475$ its Log. is 3.588101
 And $T = 42$ its Log. is 1.623249 } Add.
 $R = 0,0125$ its Log. is 8.397940 }

The Sum is the Logarithm 3.609290 of 4067,15
 And $4067,15 + 3873,475 = 7940,625$ the Dividend.

Again, $T = 42$ its Log. is 1.623249
 $T - 1 = 41$ its Log. is 1.612784 } Add.
 And $\frac{1}{2} R = 0,0125$ its Log. is 8.096910 }

The Sum is the Logarithm 1.332943 of 21,525
 And $21,525 + 42 = 63,525$ for the Divisor.

Then the Divid. 7940,625 its Log. 3.899854 } Subtr.
 The Divisor 63,525 its Log. is 1.802945 }

There remains the Logar. 2.096909 of 125
 Viz. 125 *l.* is the Half-yearly Rent, &c. As before.

Case

Case 3. When U , P , and R , are given; To find T .
That is, when any Annuity, with its proposed Value or Purchase, and the Rate of Interest per Cent. are given; Thence to find how Long the Purchaser ought to enjoy it.

To the present Worth, Add half the Annuity; then Multiply that Sum with the Ratio, and Subtract the Product from the Annuity; Then Divide the Remainder by the Product of the Annuity Multiplied with the Ratio, and call the Quotient x .

Rule. Next, Square that Quotient (viz. Multiply it with it self) and call that Square xx .

Then Divide the present Worth by the same Divisor, viz. by the Product of the Annuity into the Ratio; and to twice that Quotient Add the Square Number called xx ; Then Extract the Square Root of their Sum, and from that Root Subtract the Number called x ; The Remainder will shew the Time sought.

Example.

Suppose one would Lay out 3873 l. 9 s. 6 d. ready Money for an Annuity of 250 l. per Annum, to be paid by Half-yearly Payments; How long may he enjoy that Annuity, to be allow'd but 5 per Cent. per Annum, Simple Interest for his Money.

Or thus, which is the same thing:

In what Time will 250 l. per Annum, Pay off a Debt of 3873 l. 9 s. 6 d. by Half-yearly Payments, allowing the Creditor 5 per Cent. Interest for his Money, until the Debt be Discharged?

In this Question there is given $P=3873,475$
 $U=125$, viz. the Half-yearly Payments, and $R=1,025$, viz. the $\frac{1}{4}$ of the Ratio of 5 per Cent. Thence to find T , the Number of Half-Years.

First $\frac{1}{2}U=62,5$ and $62,5 + 3873,475 = 3935,975$

Then

Then $3935,975 \times 0,025 = 98,4$ *ferè*, and $125 - 98,4$
Leaves 26,6 for the first Dividend.

And $125 \times 0,025 = 3,125$ for the Common Divisor.

Then $3,125 \mid 26,600$ (8,512 the Number call'd x .

And $8,512 \times 8,512 = 72,454$ the Number call'd xx .

Again, the Common Divisor is 3,125

And $3,125 \mid 3873,475$ (1239,512

And twice 1239,512 is 2479,024 } Add
The Number call'd xx is 72,454 }

And the Square Root of 2551,478 is 50,512

Lastly $50,512 - 8,512$ (the Number call'd x) leaves
 $42 = T$ the Number of Half-Years ; consequently the
Half of 42, *viz.* 21 will be the Time or Number of
Years required by the Question.

This Case may also be Resolved by Logarithms,
but it is attended with some seeming Difficulties, which
requires a due Consideration of the Rule.

As for Instance in the Last Example, wherein
 $P = 3873,475$; $U = 125$; and $R = 0,025$; To find T .
 $62,5 + 3873,475 = 3935,975$ its Log. 3.595053 } Add
 $R = 0,025$ its Log. is 8.397940 }

The Sum is the Log. of 98,4 — 1.992993

$125 - 98,4 = 26,6$ its Log. 1.424882 of the *Divid.*

Again, $U = 125$ its Log. is 2.096910 } Add

And $R = 0,025$ its Log. is 8.397940 }

The Sum is the Log. of 3,125 : 0.494850 the *Divisor*.

To be Subst. from the upper Log. 0.930032 Remains the
Logarithm of 8,512 the Number called x .

And the Double of the last Log. is 1.860064 which is
the Logarithm of 72,454 the Number call'd xx .

Again,

Again, $P = 3873,475$ its Log. 3.588100 } Subtract
 And the Log. of the Divisor is $0,494850$ }
 There remains the Logarithm 3.093250 of $1239,512$
 Twice $1239,512$ is $2479,024$ } Add
 The Num. call'd xx $72,454$ }

The Sum is $2551,478$ its Log. is 3.406792

The Half of the last Log. is 1.703396 its
 Number is $50,512$ from which Subtract the Num.
 call'd $x = 8,512$

Remains $42,000 = T$ the Number of Half-Years,
 &c. As before.

Case 4. When U , P , and T , are given; To find R .

That is, when any Annuity, with the Time of its
 Continuance, and its proposed Present Worth for that
 Time are given; Thence to find what Rate of Interest
 per Cent. is allow'd the Purchaser.

Rule. $\left\{ \begin{array}{l} \text{Multiply the Annuity with the Time, and from} \\ \text{the Product Subtract the present Worth; then} \\ \text{make twice that Remainder a Dividend.} \\ \text{Next, Add the Annuity to twice the present} \\ \text{Worth, and from their Sum Subtract the first} \\ \text{Product; then Multiply the Remainder with the} \\ \text{Time, and make the Product a Divisor; by} \\ \text{which Divide the aforesaid Dividend, and the} \\ \text{Quotient will shew the Ratio of the Rate of In-} \\ \text{terest requir'd.} \end{array} \right.$

Example.

Suppose 3873 l. 9 s. 6 d. were given for an Annuity of
 250 l. per Annum; to be paid Half-yearly, and to con-
 tinue 21 Years; What Rate of Interest per Cent. &c.
 is allow'd the Purchaser.

Here is given $P = 3873,475$; $U = 125$ the Half-
 yearly Payments; and $T = 42$, the Number of Half-
 Years in 21 Years.

Then

Then $125 \times 42 = 5250$ the first Product;
And $5250 - 3873,475 = 1376,525$ its Double is $2753,05$
for the Dividend.

Next, $2P = 7746,95$ and $7746,95 + 125 = 7871,95$
and $7871,95 - 5250$ (the first Product) leaves $2621,95$

Again, $2621,95 \times 42 = 110121,9$ for a Divisor.

Then $110121,9 \div 2753,05$ ($0,025 = R$ the Ratio of
the Rate *per Cent.* for Half-yearly Payments, conse-
quently $0,025 \times 2 = 0,05$ is the Ratio of the Rate
per Cent. per Annum; And then it will be,
As $1 : \text{To } 0,05 :: \text{So is } 100 : \text{To } 5$, the Rate of
Interest as was requir'd.

The same may be done by *Logarithms*.

Thus, $U = 125$ its Log. is 2.096910 } Add
And $T = 42$ its Logar. is 1.623249 }

The Sum is the Logarithm 3.720159 of 5250

And $5250 - 3873,475 = 1376,525$ the Double where-
of is $2753,05$ its Log. is 3.439814 to be Reserv'd.

Next $2P = 7746,95$ and $7746,95 + 125 = 7871,95$
and $7871,95 - 5250$ the first Product or Num. above,
There remains $2621,95$ its Log. is 3.418624 } Add
The Time $42 = T$, its Logar. is 1.623249 }

Substra& this Logarithm 5.041873 Sum
From the Reserved Logar. viz. 3.439814

And there remains the Logarith. 8.397941 of $0,025$
Viz. $0,025 = R$ the Ratio of the Rate *per Cent.* for
Half-yearly Payments, &c. As above.

These Four Examples may be sufficient to shew
how any other of the like kind may be Resolved, ei-
ther for whole Years, or those of Quarterly Rents, &c.
provided the Purchased Rent, Lease, or Annuity is to
commence immediately.

But if it be required to find the present Value, or
Worth of any Rent, or Annuity, &c. in Reversion;
That

That is, when it is not to be Enter'd upon until after some Time or Number of Years are past.

You must first find what the proposed Annuity, or Rent, &c. would be worth for the given Time of its Continuance, as if it were to be immediately Enter'd upon:

Then < And then you must find what Principal or Sum being forborn at Interest during the Time of that Reversion, would Amount to or Raise the aforesaid Value; That Principal will be the Sum which should be paid for the proposed Annuity in Reversion.

Example.

There is the Reversion of a Lease of 175 l. per Annum, to be Lett for Eleven Years, which are to commence after Nine Years are expired; 'Tis required to find the present Worth of that Lease, allowing the Purchaser 6 per Cent. for his ready Money.

The first Work in this Question must be to find what 175 l. per Annum, to continue 11 Years, is worth in ready Money, supposing it were to be immediately Enter'd upon: As in Case 1. Page 40.

That is, here is given $U = 175$; $T = 11$; and $R = 0,06$ To find P , the Present Worth.

The Work Prepared will stand thus $11 - 1 = 10$
and $\frac{1}{2} R = 0,03$. Then $11 \times 10 \times 0,03 = 3,3$
And $3,3 + 11 = 14,3$.

Next, $14,3 \times 175 = 2502,5$, for a Dividend,

Again, $11 \times 0,06 = 0,66$ and $0,66 + 1 = 1,66$ the Divis.
Then $1,66 \overline{) 2502,5} (1507,53 = P$ the Present Worth,
if the Lease were to commence immediately, but because it is not so, Therefore

The next Work must be to find what Principal or Sum, being put out to Interest for Nine Years, at
6 per

6 per Cent. Sc. will Amount to 1507, 53 l. the which will be found (per Case 2. p. 24.)

To be 978,9162 = 978 l. 18 s. 3 $\frac{1}{4}$ d. which is the true present Worth of that Lease in Reversion, as was required.

2. Or if it be required to find what Annuity, Sc. in Reversion, may be Purchased for any proposed Sum, and at any given Rate of Interest per Cent. when the Time the Annuity is not to be Entered upon, and the Time of its Continuance are both given.

The first Work must be to find what the Sum proposed to be laid out in the Reversion, would Amount to in that Time wherein the Annuity is not to be in Possession, as if it were forborn at Interest during that Time, at any given Rate per Cent. (by Rule 1. Page 23.)

Then

And the second Work will be to find what Annuity, Sc. that Amount will purchase, (as in Case 2. Page 41.) and that will be the Answer to the Question.

These Two are the most general and useful Questions that relate to Purchasing Annuities, or Leases in Reversion: Not but that, if there be occasion, either the Time, or the Rate of Interest may be found by a due Application of their respective Rules.

I know that Sir Samuel Moreland, and several other Authors, that Treat upon the Subject of Interest and Annuities, do assert, That neither the Arrears, nor the present Worths of Annuities can be truly Computed at Simple Interest, but only at Compound Interest; And therefore, say they, all Computations of Annuities, or Leases, Sc. at Simple Interest are to be laid aside as useless.

'Tis true indeed, that in Purchasing of Annuities, or Taking of Leases, either to be in present Possession, or in Reversion; it is usual to allow the Purchaser Compound Interest for his ready Money, notwithstanding it

it be not lawful to Lett out Money at Interest upon Interest, *viz.* at Compound Interest.

However, that we may the better judge whether such Computations as are made of Annuities in Arrears at Simple Interest, be useful or no: Let us here see and compare how near they will agree with the Ninety Nine Years Annuities, settled by the late Acts of Parliament.

Those Annuities were Sold at Sixteen Years Value, or Purchase; That is, 1600 *l.* ready Money Bought 160 *l.* *per Annum*, to continue 99 Years, and so in proportion for a Greater, or Lesser Sum.

Now it may be reasonable to suppose, that, the Parliament design'd at the Settling of those Annuities at that Value, to allow the Purchaser after the Rate of Six *per Cent.* *per Annum* Simple Interest for his ready Money, according to the former Laws relating to Interest: If so, the Interest of 1600 *l.* would be 96 *l.* *per Annum* for ever, (*as by Case 1. Pag. 22*) consequently there will remain but 4 *l.* *per Annum* for 99 Years in Lieu of the Principal or Purchase-Money: That is, there will be but $99 \times 4 \text{ l.} = 396 \text{ l.}$ repaid instead of the 1600 *l.* If it be only receiv'd without any further Consideration.

But, if we Compute what that 4 *l.* *per Annum* would Amount to, if it were forborn in Arrears for 99 Years, allowing 6 *per Cent.* for every Payment as it becomes due, &c. (*per Case 1. Pag. 33*) Then will that Amount undoubtedly shew what Sum will be truly repaid for the 1600 *l.* first paid for that Annuity.

And in order to that there is given $U=4$ $T=99$ and $R=0,06$ the Ratio of the Rate of 6 *per Cent.* To find A , the Amount.

Which Prepared for the Work is $99 - 1 = 98$

And $\frac{1}{2} R = 0,03$

Then $99 \times 98 = 9702$, and $9702 \times 0,03 = 291,06$
To find A , the Amount.

F

Next

Next $291,06 + 99 = 390,06$, and $390,06 \times 4 = 1560,24$ equal *A*, viz. 1560 *l.* 4 *s.* 9½ *d.* is the Sum that will in effect be received for the 1600 *l.* over and above the 96 *l.* *per Annum* Interest.

Now in this Calculation I take for granted what I have elsewhere proved ; (*Vide Young Mathematician's Guide, Page 248*) viz. That if any Yearly Rent be forborn, or Let run in Arrears, and any Rate of Simple Interest be allow'd for every Payment as it becomes due ; the Amount, or Sum of all those Payments will be the very same, as if every Year's Rent were actually receiv'd, and immediately put out to Interest at the same Rate.

Whence it follows, That altho' 396 *l.* be all the Money that's actually received in the space of 99 Years, over, or above the 96 *l.* *per Annum* Interest ; yet according to the Nature of Interest, there is effectually 1560 *l.* 4 *s.* 9½ *d.* receiv'd ; supposing the Annuity to be paid but Yearly, which is indeed paid Quarterly ; and therefore the Sum will be still nearer to 1600 *l.* which plainly shews that the Computations of Annuities, or Rents run in Arrears at Simple Interest, are not to be wholly rejected as useless.

But I cannot pretend to say much in the behalf of such Computations as are made about the present Worth of Annuities, or Leases, &c. at Simple Interest, nor are they to be relied on in Practice : However, because this Tract should not be wanting in that Part, I thought it convenient to shew (*altho' but briefly*) how those Computations may be perform'd.

And shall in the next Chapter handle the Business of Purchasing Annuities, &c. more fully ; especially that Part of it which relates to Annuities, or Leases in Reversion.

C H A P. IV.

The Calculation of Questions in Compound Interest, and Annuities ; perform'd by the help of Logarithms.

Comound Interest is that which is produced from any Principal and its Interest put together, as the Interest of that Principal becomes due : That is, at every Payment, Or rather at the Times when the Payments become due, there is still created a New Principal, by the Increase of the growing Interest ; and therefore it is called *Interest upon Interest*, or *Compound Interest* ; which is grounded upon a Series of Geometrical Proportionals continued, as I have plainly demonstrated in the *Young Mathematician's Guide*, P. 253, &c.

In all Computations of this kind, whether they are about Money forborn at Interest, or those relating to Annuities, &c. we generally make use of the Amount or Produce of One Pound and its Interest for One Year, instead of the Ratio of the given Rate of Interest ; As before in Simple Interest.

And that Amount of 1 l. is no more but the Ratio of the given Rate Added to a Unit or 1.

For the Ratio is the Interest of 1 l. (*vid. Pag. 20*) consequently if that 1 l. be Added to it, the Sum will be the Amount of 1 l. (*per Rule at Case 1. Pag. 23*) Or

The Amount of 1 l. for One Year at any given Rate of Interest *per Cent.* may be found by this Proportion ;

Viz.

As 100 : 106 :: 1 : 1,06 the *Amount* of 1 l. at 6 *per Cent.*
Or 100 : 107 :: 1 : 1,07 the *Amount* of 1 l. at 7 *per Cent.*
And so on for any other given Rate of Interest.

Or the aforesaid *Amounts* of 1 l. may be otherways found by *Logarithms* ;

F 2

Thus

Thus, $\left\{ \begin{array}{l} \text{Add the proposed Rate of Interest to 100, and} \\ \text{from the Logarithm of that Sum, Subtract the} \\ \text{Logarithm of 100 (viz. 2.000000) the Re-} \\ \text{mainder will be the Logarithm of the Amount} \\ \text{of 1 l. at that Rate of Interest per Cent.} \end{array} \right.$

Example.

Suppose the given Rate of Interest to be that of 6 per Cent. per Annum.

Then $100 + 6 = 106$ its Log. is 2.025306 } Subst.
And the Log. of 100 is 2.000000 }

There Remains the Logarithm 0.025306 of 1,06
Viz. 1,06 is the Amount of 1 l. at 6 per Cent. &c.
And so for any other Rate of Interest.

And because 'tis the Logarithms of those Amounts of 1 l. that are of greatest Use in the following Calculations, I have here annexed a small Table of several Rates of Interest, with the Amounts of 1 l. per Annum at those Rates, and the Logarithms of those Amounts.

Rates of Interest per Cent. l.	The Amounts of 1 l. Viz. R.	Logarithms of those Amounts, Viz. of R.	Rates of Interest per Cent. l.	The Amounts of 1 l. Viz. R.	Logarithms of those Amounts, Viz. of R.
3	1,03	0.012837	7	1,07	0.029384
4	1,04	0.017033	$7\frac{1}{2}$	1,075	0.031408
$4\frac{1}{2}$	1,045	0.019116	8	1,08	0.033424
5	1,05	0.021189	$8\frac{1}{2}$	1,085	0.035430
$5\frac{1}{2}$	1,055	0.023252	9	1,09	0.037426
6	1,06	0.025306	$9\frac{1}{2}$	1,095	0.039414
$6\frac{1}{2}$	1,065	0.027350	10	1,1	0.041393

These things being understood, you may proceed to the Resolving of Questions about Money forborn at Compound Interest.

Set.

**Sect. I. Of Compound Interest, Or Money
Lett out at Interest upon Interest.**

I shall here make use of the same Letters to denote the several Parts of the Question, as before in *Page 22.* except *R* only.

P. Denotes any Principal put to Interest.
Viç. $\left\{ \begin{array}{l} t. \text{ The Time of its Continuance at Interest.} \\ A. \text{ The Amount or Principal and its Interest.} \end{array} \right.$
 And *R.* Denotes the Amount of 1 *l.* for One Year.
 That is, the Ratio of the given Rate more 1.

Then it will be, $PR^t = A.$

This Equation admits of the same Four Cases, or Variety of Questions, as that of Simple Interest in *Page 22.* And here it may not be amiss to make use of the same Examples, that we may the more readily see the Difference that is between Simple Interest, and Compound Interest.

Case 1. If *P*, *t*, and *R*, are given; To find *A* the Amount.

Rule. $\left\{ \begin{array}{l} \text{First Multiply the Log. of } R, \text{ the Amount of} \\ \text{1 } l. \text{ at the given Rate per Cent. with the Time,} \\ \text{Then to that Product Add the Log. of the pro-} \\ \text{posed Principal, and their Sum will be the Log.} \\ \text{of the Amount required.} \end{array} \right.$

Example.

What Sum will 567 l. 10 s. Amount to in Nine Years, at the Rate of 6 per Cent. per Annum?

Hence *R* = 1,06 its Log. 0.025306 } Multiply
 And *t* = 9 }

Their Product is 0.227754 } Add
 Also *P* = 567,5 its Log. is 2.753966 }

The Sum is the Logarithm. 2.981720 of 958,78 = *A*

That is, 958 *l.* 15 *s.* 7 *d.* will be the Amount requir'd:
 F 3 Conse-

Consequently 958l. 15s. 7d. = 567l. 10s. = 391l. 5s. 7d. will be the Interest only, which is more than the Simple Interest of that Principal, for the same Time, by 84l. 16s. 7d. See Page 23.

Case 2. When A , t , and R , are given ; To find P .

Rule. { Multiply the Log. of R the Amount of 1 l. at the given Rate per Cent. with the Time (as before ;) Then Subtract that Product from the Logar. of the proposed Amount, and there will Remain the Logarithm of the Principal required.

Example.

What Principal or Sum of Money, being put out to Interest for Nine Years, will Amount to (or Raise the Sum of) 958 l. 15 s. 7 d. at the Rate of 6 per Cent. per Annum?

Or thus ; There is a Debt of 958 l. 15 s. 7 d. which is not due until Nine Years hence, but 'tis agreed to be paid in present Money ; What Sum must the Creditor Receive, allowing the Rebate or Discompt, of 6 per Cent. per Annum to the Debtor for his ready Money.

Here $A = 958,78$ its Log. 2.981720

And $R = 1,06$ its Log. is 0.025306 } Multiply.
Also $t = 9$

Subst. this Log. from the first 0.227754

There Remains the Logar. 2.753966 of 567,5 = P .

That is, 567 l. 10 s. is the Principal (or ready Money) as was required.

Case 3. Suppose P , t , and A , are given ; To find R .

Rule. { From the Log. of the proposed Amount, Subtract the Log. of the proposed Principal ; Then Divide the Remainder by the Time, and the Quotient will be the Logarithm of R , the Amount of 1 l. &c. As in this Example.

Exam-

Example.

At what Rate of Interest per Cent. &c. will 567 l. 10 s. Raise a Stock, or Amount to 958 l. 15 s. 7 d. in Nine Years Time?

Here is given $A=958,78$ its Log. 2.981720 } Subtract
And $P=567,5$ its Logarithm is 2.753966 }

Also $t=9$ Then $9) 0.227754$ (0,025306
the Log of $1,06=R$; then $1,06-1=0,06$ the Ratio,
And it will be. As $1 : 0,06 :: 100 : 6$ the
Rate of Interest per Cent. &c. (See Page 25.)

Case 4. Having P , R , and A , given; To find t .

Rule. { From the Log. of the proposed Amount, Subtract the Log. of the proposed Principal (as before) and Divide their Difference by the Log. of R the Amount of 1 l. at the given Rate, and the Quotient will shew the Time required.

Example.

In what Time will 567 l. 10 s. Amount to, (or Raise a Stock of) 958 l. 15 s. 7 d. at the Rate of 6 per Cent. per Annum?

Here is given $A=958,78$ its Log. 2.981720 } Subst.
And $P=567,5$ its Logarithm is 2.753966 }

Also $R=1,06$ its Log. is 0,025306) 0.227754 ($9=t$
viz. Nine Years. will be the Time required by the Question.

These Four Examples are sufficient to shew how all Questions of the like kind may be truly Resolved; if the Time given (or sought) be whole Years; but if it be not, then the Odd Time, whether it be under, or above a Compleat Year, must be Reduc'd into Decimal Parts of a Year, (See pag. 27) by help of the two Tables in Page 28 and 29, &c. and then it will be as easie to Resolve any of the foregoing Cases, when the

the Time given (or sought) is in Parts of a Year, as it is for those in whole Years.

Example 1. When the Time is Less than a Year. Let it be required to find the Amount, of 375 l. for 210 Days, at 6 per Cent. &c.

In this Question there is given, $P=375$ $R=1,06$ and $t=210$ Days $=0,5753$ Decimal Parts of a Year, per Table, pag. 29; Thence to find A ; as in Case 1. of this Section.

First $R=1,06$ its Log. is 0.025306
And $t=$ 0.5753 } Multiply.

Their Product is 0.014558
Again $P=375$ its Log. is 2.574031 } Add

The Sum is the Logar. 2.588589 of $387,78=A$.
That is 387 l. 15 s. 7 d. will be the Amount requir'd.

Example 2. Suppose it were requir'd to find what 265 l. would Amount to in Five Tears and 135 Days; at the Rate of 6 per Cent. &c.

In this Question is given $P=265$ $R=1,06$ and $t=5,3698$ To find A the Amount; As before.

Thus $R=1,06$ its Log. 0.025306
And $t=$ $5,3698$ } Multiply

Their Product is 0.135888
Again $P=265$ its Log. 2.423246 } Add

The Sum is the Logar. 2.559134 of $362,355=A$.
Viz. 362 l. 7 s. 1 d. will be the Amount or Sum requir'd.

Example 3. Let it be requir'd to find what Principal, or Sum put to Interest at 6 per Cent. will Amount to (or Raise a Stock of) 362 l. 7 s. 1 d. in Five Tears and 135 Days.

Here is given $A=362,355$ $R=1,06$ and $t=5,3698$
To find P , as in Case 2.

Thus

Thus $A=362,355$ its Log. 2.559134 of the Divid.

And $R=1,06$ its Log. is 0.0253067
 $t=5,3698$ } Multiply

Then Subtract this Logar. 0.135888 from 1st Log.

And there Remains the Log. 2.423246 of $265 = P$.

Viz. 265 l. will be the Principal, or Sum requir'd.

If the Work of these Three Examples be well understood, it must needs be easie to find proper Answers to all Questions that can be proposed in the other Cases, *viz.* for any proposed Time, and Rate of Interest, according as the Date requires.

Sect. 2. Of Annuities, or Rents, &c. that are in Arrears, Computed at Compound Interest.

I shall here make use of the same Letter to represent the same things as in Page 32. Save only that R doth here denote the Amount of 1 l. As in the last Section.

Viz. $\left\{ \begin{array}{l} U = \text{the Annuity, or Rent.} \\ t = \text{the Time of its being unpaid.} \\ A = \text{the Amount, or Sum of all the Arrears.} \end{array} \right.$
 And $R = \text{the Amount of 1 l. \&c. as in Page 53.}$

Then will $A - U = AR - UR^t$.

From this Equation is deduced the Four following Cases, and the Rules by which they are Resolved.

N. B. The following Cases admits of some Variety, in respect to the true Intervals of the Times, which are betwixt the several Payments; *viz.* it must be well considered whether the Payments are to be made Yearly, or at the End of some part of a Year; As the $\frac{1}{2}$. $\frac{1}{4}$. $\frac{1}{12}$. $\frac{1}{365}$. That is, Half-yearly, Quarterly, Monthly, &c. the which must be very carefully minded when the Question is proposed, in any of the following Cases, or the Answers will not be true. Case

Case 1. Having U , t , and R , given; To find A .

That is, If any Annuity, or Rent, &c. with the Time of its being unpaid, and the Rate of Interest *per Cent.* are given; Thence to find what Sum all those Arrears will Amount to in that Time, allowing Compound Interest for every particular Payment, as it becomes due.

Rule. { Multiply the Log. of R (the Amount of 1 l.) with the Time; and to that Product Add the Log. of the Annuity; Then find the Number which belongs to that Sum, and Subtract the Annuity from that Number; (So far the Work is general to all Times of Payment.) Then, if the Remainder (*) be Divided by $R - 1$ (viz. by the Ratio of the Rate of Interest *per Cent.*) the Quotient will shew the Sum, or Amount of all the Arrears for Yearly Payments.

Example 1.

Suppose 356 l. Annuity, or Yearly Rent, were forborn or unpaid Nine Years; What would the Sum of all those Arrears Amount to at 6 per Cent. &c. ?

Here is given $U=356$ $t=9$ and $R=1.06$ To find A .

Thus $R=1.06$ its Log. is 0.025306 } Multiply
 $t=9$ 9

The Product is 0.227754 } Add
 Again $U=356$ its Log. is 2.551450

The Sum is the Logarithm 2.779204 of 601,4545.

And $601,4545 - 356 = 245,4545$; $R - 1 = 0.06$

Then 0.06 $245,4545$ (4090,91 = $A = 4090$ l. 18s. 2½d. being the Sum of all the Arrears for Yearly Payments.

Note, Here we may observe the Difference that is betwixt the Arrears Computed at Simple Interest, and at Compound Interest; See the same Example in Page 33.
 Or,

Or, the Amount may be also found by Logarithms without the last Division ; for having once form'd the Dividend, and Divisor (as before) then the Difference of their Logarithm will shew the Amount.

Thus 245,4545 its Log. 2.389972 } Subtract
And $R-1=0,06$ its Log. 8.778151 }

There Remains the Log. 3.611821 of 4090,91 = A .
And so on, As before.

But if the Payments are to be made at any Time Less than a compleat Year ; As $\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{12} \cdot \&c.$

Then { You must take the like part of the aforesaid Remainder (*) and make it a new Dividend : And you must also take the same part of the Logarithm of R ; and the Number answering to that part being made less by an Unit or 1, will be a new Divisor (instead of $R-1$) by which the new Dividend being Divided, the Quotient will shew the Sum of all the Arrears required.

Example 2. In Half-yearly Payments.

Suppose the aforesaid 356 l. per Annum, were to be paid by Half-yearly Payments, viz. 178 l. every Half-Year, and $\&$ were unpaid Nine Years (as before) What Sum would all those Arrears Amount to then, at 6 per Cent. &c. ?

Here (as bef.) $R=1,06$ its Log. 0.025306 } Multiply
And $t=9$ }

Their Product is 0.227754 } Add
Also $U=356$ its Log. is 2.551450 }

The Sum is the Logarithm 2.779204 of 601,4545 and $601,4545 - 356 = 245,4545$. So far the Work is just the same as before for Whole Years :

But here the $\frac{1}{2}$ of 245,4545 = 122,72725 is the New Dividend : Next,

Next, $R = 1,06$ the $\frac{1}{2}$ of its Log. is $0,012653$ whose Number is $1,029563$ and $1,029563 - 1 = 0,029563$ for the New Divisor ;

Then $0,029563) 122,72725 (4151,38 = A.$

That is, $4151\text{ l. } 7\text{ s. } 7\text{ d.}$ is now the Amount or Sum of all the Arrears; which is, $60\text{ l. } 9\text{ s. } 4\frac{1}{2}\text{ d.}$ more than the Amount in the last Example for whole Yearly Payments.

Here also the Amount may be obtain'd by Logarithms without the last Division; having once form'd the New Dividend, and its proper Divisor as before:

Thus, $122,727$ its Log. is $2,088938$ } Subtract
And $0,029563$ its Log. is $8,470748$ }

There Remains the Log. $3,618190$ of $4151,38 = A$
Viz. $4151\text{ l. } 7\text{ s. } 7\text{ d.} = A$ the Amount, &c. As before.

Example 3. In Quarterly Payments.

Let it be required to find the Amount, or Sum of all the Arrears that would be due from 245 l. per Annum, to be paid by Quarterly Payments (viz. $61\text{ l. } 5\text{ s.}$ per Quarter) supposing it to be forborn, or unpaid Ten Years and a Half, at $5\frac{1}{2}$ per Cent. &c.

Here is given $R = 1,055$ its Log. $0,023252$ } Multiply
And $t = 10,5$ }

The Product is $0,244146$ } Add
Again $U = 245$ its Log. is $2,389166$ }

Their Sum is the Logarith. $2,633312$ of $429,846$
And $429,846 - 245 = 184,846$ So far the Work is still the same as in whole Years.

But now it must be the $\frac{1}{4}$ of $184,846 = 46,2115$ for the New Dividend.

Next, $R = 1,055$ the $\frac{1}{4}$ of its Log. is $0,005813$ whose Numb. is $1,01348$ and $1,01348 - 1 = 0,01348$ for the New Divisor.

Then

Then $0,01348 \times 46,2115 = 3428,15 = A$.

That is, 3428 l. 3 s. will be the Amount or Sum of all those Arrears, as was required.

Or the last Division may be avoided, and the Amount may be found by the Logarithms; Thus,

Dividend 46,2115 its Log. 1.664750 } Subtract
Divisor is 0,01348 its Log. 8.129690 }

There Remains the Logar. 3.535060 of 3428,15 = A
Viz. 3428 l. 3 s. is the Amount requir'd, as above.

The same may be done for Monthly, Weekly, or Daily Payments, Care being taken in forming the New Dividend, and Divisor, according to their respective Time.

Case 2. When A , t , and R , are given; To find U .
That is, To find what Annuity, or Yearly Rent being forborn or unpaid, any assigned Time, will Amount to a proposed Sum, allowing any given Rate of Interest per Cent. for every Payment as it becomes due.

Note, This Case, as well as the Last, admits of some Variety in respect to the Times of Payment; viz. Whether they are Yearly, Half-yearly, or Quarterly, &c. the which must be carefully observed, as the following Rule directs.

Rule. { Multiply the Log. of R (the Amount of 1 L.) with the given Time, and the Number answering to their Product being made less by 1, will be the Divisor, (for all Times of Payment at that Rate of Interest.)
Next, Multiply the proposed Amount or Sum, with $R - 1$, viz. with the Ratio of the given Rate; Then Divide their Product by the aforesaid Divisor, and the Quotient will shew the Annuity, or Rent for Yearly Payments.

Example 1.

What Annuity, or Yearly Rent, being forborn or unpaid Nine Years, will Amount to (or Raise a Stock of) 4090 l. 18 s. 2½ d. at the Rate of 6 per Cent. for every Payment as it becomes Due?

Here is given $A = 4090,91$ $t = 9$, and $R = 1,06$
To find U .

First $R = 1,06$ its Log. is 0.025306
And $t = 9$ } Multiply

The Product is the Logar. 0.227754 of $1,68948$
And $1,68948 - 1 = 0,68948$ for the Divisor.

Again $A = 4090,91$ } Multiply
And $R - 1 = 0,06$

The Product is $245,4546$ for the Dividend.

Then $0,68948 \times 245,4546$ ($356 = U$).
That is 356 l. will be the Annuity, or Yearly Rent,
as was requir'd to be paid by Yearly Payments.

Or having found the Divisor (*as above*) the rest of
the Work may be perform'd by Logarithms only;

Thus, $A = 4090,91$ its Log. 3.611821 } Add
And $R - 1 = 0,06$ its Log. 8.778151

Sum is 2.389972 } Subtract
The Divis. is $0,68948$ its Log. 9.838522

There Remains the Logarithm. 2.551450 of $356 = U$.

Viz. 356 l. will be the Yearly Annuity, or Rent, &c.

But if the Payments are required to be paid either
Half-yearly, or Quarterly, &c. As in the last Case;

Then *Instead of $R - 1$, you must take the respective Part of the Logarithm of R (viz. of the Amount of 1 l.) and with the Number answering to that Part, being made less by 1, Multiply the proposed Amount; Then Divide that Product by the aforesaid Divisor, and the Quotient will shew what Annuity, or Rent must be paid at the proposed Time of each Payments.* Exam-

Example 2. In Half-yearly Payments.

What Annuity, or Rent, to be paid by Half-yearly Payments, being forborn or unpaid Nine Years, will Amount to the Sum of 4151 l. 7 s. 7 d. allowing 6 per Cent. for every Payment as it becomes Due?

Here is given $A = 4151,38$ $t = 9$, and $R = 1,06$
To find U .

First $R = 1,06$ its Log. is $0,025306$
And $t = 9$ } Multiply

The Product is the Log. $0,227734$ of $1,68948$
And $1,68948 - 1 = 0,68948$ for the Divisor, as before in the last Example.

But instead of the Multiplier $R - 1$, it must be $R = 1,06$ the $\frac{1}{2}$ of its Log. $0,012653$ its N^o. is $1,029563$ and $1,029563 - 1 = 0,029563$ the Multiplier.

And it must be $A = 4151,38$
Multiplied with $0,029563$

The Product is $122,72724$ the New Dividend.
Then $0,68948 \mid 122,72724$ ($178 = U$, viz. 178 l. will be the Half-yearly Payments of the Annuity, or Rent required.

Or having found the Divisor, and the New Multiplier used instead of $R - 1$; Then the Work may be done by the Logarithms,

Thus, $A = 4151,38$ its Log. $3,618190$ } Add
Multipl. is $0,029563$ its Log. $8,470749$ }

Sum $2,088939$ } Subtract
Divisor is $0,68948$ its Log. is $9,838520$ }

There Remains the Logarith. $2,250419$ of $178 = U$
Viz. 178 l. is the Half-yearly Payments, &c. As before.

Example 3. In Quarterly Payments.

Let it be required to find what Annuity, or Rent to be paid by Quarterly Payments, being forborn or unpaid

Ten Years and a Half, would Amount to the Sum of 3428l. 3 s. at Five and a Half per Cent. per Annum, for every Payment as it becomes Due?

Here is given $A = 3428,15$ $t = 10,5$ and $R = 1,055$
To find U , the Quarterly Payments.

First $R = 1,055$ its Log. is 0.023252 } Multiply
And $t = 10,5$ }

The Product is the Logar. 0.244146 of $1,75447$

Then $1,75447 - 1 = 0,75447$ the Divisor.

Again $R = 1,055$ the $\frac{1}{4}$ of its Log. is 0.005813 its Number is $1,01348$ and $1,01348 - 1 = 0,01348$ the Multiplier.

Next $A = 3428,15$
Multiplied with $0,01348$

Produces $46,21146$ the Dividend.

Then $0,75447 \mid 46,21146$ ($61,25 = U$.)

That is, 61 l. 5 s. will be the Quarterly Rents or Payments; consequently $61,25 \times 4 = 245$ l. per Annum will be the Annuity requir'd.

Or by *Logarithms*, Thus

$A = 3428,15$ its Log. is 3.535060 } Add
And $0,01348$ its Log. is 8.129690 }

Sum is 1.664750 } Subtract
Again $0,75447$ its Log. 9.877642 }

There Remains the Log. 1.787108 of $61,25 = U$
Viz. 61 l. 5 s. will be the Annuity, or Rent to be paid every Quarter, &c.

The like may be done in finding the Monthly, Weekly, or Daily Annuities, care being taken in Forming the New Multipliers, and Divisors proper for the Time given.

Case 3. Having U , A , and R given; To find t .
That is, To find the Time in which any proposed Annuity

nuity, or Rent will Amount to any assign'd Sum, at any given Rate of Interest *per Cent.* &c.

Rule. $\left\{ \begin{array}{l} \text{Multiply the proposed Amount with } R - 1, \\ \text{viz. with the Ratio of the given Rate; and} \\ \text{Divide that Product by the Annuity, and to the} \\ \text{Quotient Add 1. Then if the Logarithm of} \\ \text{that Sum be Divided by the Logarithm of } R \\ \text{(viz. the Amount of 1 l.) the Quotient will} \\ \text{shew the Time for Yearly Payments.} \end{array} \right.$

Example 1.

In what Time will 356 l. Yearly Rent, or Annuity, Amount to the Sum of 4090 l. 18 s. 2½ d. at the Rate of 6 per Cent. &c.

Here is given $U=356$, $A=4090,91$ and $R=1,06$
To find t .

Thus, $A = 4090,91$ } Multiply
And $R - 1 = 0,06$ }

$U = 356$) 245,4546 (0,68948

And $0,68948 + 1 = 1,68948$ its Log. is 0,227754

Then $R = 1,06$ its Log. 0,025306) 0,227754 (9 = t .
Viz. 9 Years will be the Time requir'd in the Question;

Or otherwise by *Logarithms*,

Thus $A = 4090,91$ its Log. is 3,611821 } Add
And $R - 1 = 0,06$ its Log. 8,778151 }

Sum 2,389972 } Subtract
Again $U = 356$ its Log. is 2,551450 }

There Remains the Logar. 9,838522 of 0,68948

And $0,68948 + 1 = 1,68948$ its Log. 0,227754. Then
as before, $1,06$ its Log. 0,025306) 0,227754 (9 = t .
Viz. 9 Years will be the Time required, if the Payments are to be paid but once a Year.

But if they are to be paid either *Half-yearly*, or *Quarterly*, &c. As before in the two last Cases;

the Value of R , (as in the *Young Mathematician's Guide*, Pag. 268.) which cannot otherwise be found by any General Rule, that can be prescrib'd here, as in the precedent Cases, but only by Approaching to it with Trials. And in order to perform that Approximation with the least Trouble, I shall here propose the following Method, as the easiest I can at present think of.

Which is thus :

First Divide the proposed Amount, by the Annuity, and Reserve the Quotient. Next Subtract 1 from that Quotient, and call the Remainder x .

Then you must make choice (by guess) of such a Number for the Value of R , as you think will be the Amount of 1 l. at the Rate of the Interest sought; and Multiply that Number into the reserved Quotient; Also you must Multiply the Logarithm of that supposed Number, with the Time; and find the Number which belongs to their Product: Then, if the Difference between that Number and the aforesaid Product of R into the Reserved Quotient, be Equal to (viz. be the same with) the Number call'd x , you have hit upon the true Value of R . But if they are not Equal, then another Trial must be made, &c. As in the following Examples.

Example 1. In Yearly Payments.

There is an Annuity, or Rent of 356 l. per Annum, and it has been forborn or unpaid Nine Years; And there is 4090 l. 18 s. 2½ d. given for the Amount, or Sum of all the Arrears; it's required to find what Rate of Interest per Cent. is allow'd for each Payment as it became Due?

Here is given $U = 356$, $t = 9$, and $A = 4090,91$.
To find R , the Amount of 1 l.

Thus $356 \mid 4090,91$ (11,49132 the Quotient to be Reserv'd; from which Subst. 1, and it will be 10,49132, which is the Number call'd x : And so far the Work will

will always be certain, it being only that of the first Side of the Equation.

Next, I will suppose the Value of R to be 1,05 and then the Reserved Quotient $11,49132 \times 1,05$ the Product will be 12,065886.

Again, If $R = 1,05$ its Log. 0.021189 } Multiply
And $t = 9$

The Product is the Logar. 0.190701 of 1,55132
Then $12,065886 - 1,55132 = 10,51456$ which is More than 10,49132 the Number called x ; Therefore this supposed Value of $R = 1,05$ is taken too Little.

Again, For a second Trial I will suppose $R = 1,07$ and then the Reserved Quotient $11,49132 \times 1,07$ the Product will be 12,29571.

And if $R = 1,07$ its Log. is 0.029384 } Multiply
And $t = 9$

The Product is the Logar. 0.264456 of 1,83847
Then $12,29571 - 1,83847 = 10,45724$ which is Less than 10,49132 the Num. called x ; And therefore I conclude the Last $R = 1,07$ was taken too Big, and that the Value of R is some Num. between 1,05 and 1,07

Let us therefore take $R = 1,06$ (*viz. in the Middle between 1,05 and 1,07.*) And then the Reserved Quotient $11,49132 \times 1,06$, the Prod. will be 12,180799

And if $R = 1,06$ its Log. is 0.025306 } Multiply
And $t = 9$

The Product is the Logar. 0.227754 of 1,689479
Then $12,180799 - 1,689479 = 10,49132$ which is just the same with the Number called x .

Whence I conclude that the true Value of $R = 1,06$ and $1,06 - 1 = 0,06$ is the Ratio of the Rate of Interest sought.

Then it will be, As 1 : Is to 0,06 : : So is 100 : To 6.
the Rate of Interest *per Cent.* As was requir'd for Yearly Payments. But

But if the Payments are to be made either Half-yearly, or Quarterly, &c.

Then { Instead of Multiplying the Reserved Quotient, with the supposed Value of R (as above) you must take such a Part of the Log. of that R (viz. of the assumed Amount of 1*l.*) as the given Time requires, and with its Number Multiply the aforesaid Reserv'd Quotient, and then proceed on as before.

Example 2. In Half-yearly Payments.

Suppose the same Annuity of 356*l.* per Annum were to be paid Half-yearly (viz. 178*l.* every Half-Year) If it be forborn Nine Years, and then Amounts to the Sum of 4151*l.* 7*s.* 7*d.* What Rate of Interest per Cent. is allow'd, &c.?

Here is given $U = 178$ (viz. the $\frac{1}{2}$ of 356*l.*) $t = 9$ and $A = 4151,38$ To find R , the Amount of 1*l.* &c. First 178) 4151,38 (23,32236 the Reserved Quot. and $23,32236 - 1 = 22,32236$ the Number called x . So far is the same as before in whole Years.

Now here I will assume the first Value of $R = 1,06$ Then if $R = 1,06$ the $\frac{1}{2}$ of its Log. 0,012653 and the Number to that Half Log. is 1,029563 the Multiplier instead of $R = 1,06$ as before: And then the Reserved Quotient $23,32236 \times 1,029563$, the Product will be 24,011839.

Again, If $R = 1,06$ its Log. is 0,025306 } And $t = 9$ } Multiply

Their Product is the Logar. 0,227754, of 1,689479

Then $24,011839 - 1,689479 = 22,32236$ which is just the same with 22,32236 the Number called x . And therefore the true Value of $R = 1,06$ And so on for the Rate per Cent. as in the last Example.

Example

Example 3. In Quarterly Payments.

An Annuity of 245 l. per Annum, to be paid Quarterly (viz. 61 l. 5 s. every Quarter) is forborn Ten Years and a Half; and then 'tis said to Amount to the Sum of 3428 l. 3 s. It is required to find what Rate of Interest per Cent. is allow'd; &c.

Now here is given $U = 61,25$ (viz. the $\frac{1}{4}$ of 245 L.)
 $t = 10,5$ and $A = 3428,15$ To find R , the Amount of 1 l. &c.

First $61,25 \times 3428,15 = 55,96979$ the Reserved Quot.
 and $55,96979 - 1 = 54,96979$ the Numb. called x .

Then let us here suppose $R = 1,05$ and then the $\frac{1}{4}$ of its Log. will be 0.003297 whose Num. is 1,01227 the Multiplier.

And the Reserved Quotient $55,96979 \times 1,01227$
 the Product will be 56,65754.

Again, If $R = 1,05$ its Log. is 0.021189 } Multiply
 And $t = 10,5$

The Product is the Logarithm 0.222484 of 1,66911

Then $56,65754 - 1,66911 = 54,98843$ which is a little More than 54,96979 the Number called x . Therefore the Value of $R = 1,05$ is taken something too Little.

And making a second Trial, by supposing $R = 1,06$ and then proceeding on as before, I find the Result to be about as much Less than 54,96979 the Number called x , as the last found Number was too much.

I shall therefore assume $R = 1,055$ viz. in the Middle between 1,05 and 1,06.

And then if $R = 1,055$ the $\frac{1}{4}$ of its Log. 0.005813 and its Number is 1,01348 for the Multiplier.

Then the Reserved Quotient $55,96979 \times 1,01348$
 the Product is 56,72426.

Again,

Again, If $R=1,055$, its Log. is 0.023252 } Multiply
 And $t = 10,5$ }

The Product is the Logarithm 0.244146 of $1,75447$
 Then $56,72426 - 1,75447 = 54,96979$ which is
 exactly the same with the Number called x .

And therefore I conclude that the true Value of
 $R=1,055$ and $1,055 - 1 = 0,055$ the Ratio of the
 Rate. And

Then it will be As $1 : 1$ is to $0,055 ::$ So is $100 : 5,5$,
Viz. Five and a Half is the Rate of Interest *per Cent.*
per Annum, as was required.

What hath been here done by Multiplication, and
 Division, may, if you please, be perform'd by Loga-
 rithms, as in the precedent Cases.

As for Instance in the last Example ;

Wherein $A=3428,15$ its Log. 3.535060 } Subtract
 And $U=61,25$ its Log. is 1.787108 }

The Remainder is the Log. 1.747952 of $55,9698$
 which is here the Reserved Quotient.

And $55,9698 - 1 = 54,9698$ the Number called x .

Then if $R=1,885$ the $\frac{1}{2}$ of its Log. 0.005813 } Add
 the Log. of the Reserv'd Quotient 1.747952 }

The Sum is the Logarithm 1.753765 of $56,7243$

Again, If $R=1,055$ its Log. is 0.023252 } Multiply
 And $t = 10,5$ }

The Product is the Logarithm 0.244146 of $1,75447$

Then $56,7243 - 1,75447 = 54,96983$ is just the
 same with the Number called x ; And therefore, &c.
 As above.

Self.

Sect. 3. To find the Present Worth of Annuities, Pensions, or Leases, &c. at Compound Interest.

I shall here make use of the same Letters to represent the several Parts of the Question, as in *Sect. 3. Pag. 39.* And it may not be amiss to make use also of the same Examples as are in that Section; save only that I shall here Compute them for whole Years, Half-Years, and Quarterly Payments, as in the last Section.

Let $\left\{ \begin{array}{l} U. \text{ Denote the Annuity, or Rent.} \\ t. \text{ The Time of its Continuance.} \\ R. \text{ The Amount of 1 l.} \end{array} \right. \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{As before;}$
And $P.$ The Present Worth of the Annuity, &c.

$$\text{Then will} \quad PR^t = \frac{UR^t - U}{R - 1}.$$

(Vide the Young Mathematician's Guide, Page 269.)

From this Equation is deduc'd the following Rules, which also admit of Four Cases.

Case 1. Having U , t , and R , given; To find P
That is, If any proposed Annuity, or Yearly Rent, and the Time of its Continuance are given; To find the Present Worth of that Annuity, allowing any Rate of Compound Interest per Cent. to the Purchaser, &c.

Rule. $\left\{ \begin{array}{l} \text{Multiply the Log. of } R \text{ (the Amount of 1 l.)} \\ \text{with the given Time; and from the Number} \\ \text{which belongs to that Product, Subtract 1: Then} \\ \text{Multiply the Remainder with the Annuity, and} \\ \text{the Product will be a Dividend.} \\ \text{Then Multiply the Number which belong'd to} \\ \text{the first Product, with } R - 1 \text{ (viz. with the} \\ \text{Ratio of the Rate) and make their Product a Di-} \\ \text{visor; The Quotient arising from thence will shew} \\ \text{the Present Worth for Yearly Payments.} \end{array} \right.$

H

Example

Example 1. In Yearly Rents.

Suppose a Lease of 250 l. per Annum, were to be Lett for 21 Years ; What may the present Worth of that Lease be, at the Rate of 5 per Cent. &c. ?

In this Question there is given $U = 250$, $t = 21$, and $R = 1,05$ To find P , the present Worth for Yearly Rents.

First $R = 1,05$ its Log. is 0.021189 } Multiply
And $t = 21$ }

The Product is the Logar. 0.444969 of $2,7859$
Then $2,7859 - 1 = 1,7859$ And $1,7859 \times 250 = 446,475$
for the Dividend.

Again $R - 1 = 0,05$ And $2,7859 \times 0,05 = 0,139295$
for the Divisor.

Then $0,139295 \mid 446,475$ ($3205,2478 = P$).
That is, 3205 l. 5 s. will be the present Worth of such a Lease as was required.

Or the same may be otherwise performed by *Logarithms* : Thus,

First $R = 1,05$ its Log. is 0.021189 } Mult. as before.
And $t = 21$ }

The Product is 0.444969 } Add
Again $U = 250$ its Log. 2.397940 }

The Sum is the Logar. 2.842909 of $696,48$ from which Subtract the Annuity 250 . and there will Remain $446,48$ its Logat. is 2.649802

The Log. of R , Mult. with t , is 0.444969 } Add
And $R - 1 = 0,05$ its Log. is 8.698970 }

From the up. Log. take this Log. 9.143939

There Remains the Logarith. 3.505863 of $3205,25$
That is 3205 l. 5 s. will be the present Worth requir'd by the Question, as before ; supposing the Rents to be paid but once a Year.

But

But if the Payments of the Annuity, or Rents, &c. are to be made at any Time Less than a compleat Year, As $\frac{1}{2}$. $\frac{1}{4}$. $\frac{1}{12}$ &c. That is, *Half-yearly*, or *Quarterly*, &c. Then the Divisor must be otherwise form'd than by $R - 1$.

Thus { You must take such a part of the Log. R the Amount of 1 l. as the given Time requires, and make use of the Number answering to that Part, made less by 1, instead of $R - 1$ the Ratio, &c. As in the following Examples.

Example 2. For Half-yearly Rents.

Suppose the aforesaid 250 l. per Annum, were to be paid Half-yearly (viz. 125 l. every Half-year) and a Lease of it were to be Lett for 21 Years; What would the present Worth of that Lease be, at the Rate of Five per Cent. &c.

Here is given $U = 125$, $t = 21$, and $R = 1,05$ To find P .
First $R = 1,05$, its Log. is .021189 } Multiply
And $t = 21$

The Product is the Logarith. 0.444969 of 2,7859
Then $2,7859 - 1 = 1,7859$ and $1,7859 \times 125 = 223,2375$
for a Dividend. Which is only the Half of the Dividend in the last Example.

Next $R = 1,05$ the $\frac{1}{2}$ of its Log. is 0.010594 whose Number is 1,0247 and $1,0247 - 1 = 0,0247$ to be the Multiplier instead of $0,05 = R - 1$.
That is, $2,7859 \times 0,0247 = 0,06881$ will be the Divis.

Then $0,06881 \mid 223,2375$ ($3244.25 = P$.
Viz. 3244 l. 5 s. will be now the present Worth of that Lease, which is more by 39 l. than that in the last Example for Yearly Payments.

And by the way, we may here observe the great Difference that is betwixt the present Worth of any Annuity, or Lease, &c. Computed at Simple Interest,
H 2 and

and the same at Compound Interest. See the Example in Page 40.

The same may also be performed by *Logarithms* as in the last Example, having found the Multiplicator instead of $R - 1$, As above.

Then $R = 1,05$ its Log. is $0,021189$ } Multiply
And $t = 21$ }

Again $U = 125$ its Log. is $2,096910$ } Add
 $0,444969$ }

The Sum is the Logarithm $2,541879$ of $348,24$
From this $348,24$ Subtract the Rent 125 , and there
Remains $223,24$ its Logarith. is $2,348772$

The Log. of R , Multip. with t , is $0,444969$ } Add
The new Multip. $0,0247$ its Log. $8,392697$ }

From the up. Log. Subst. this Log. $8,837666$

There Remains the Logarithm $3,511106$ of $3244,2$
Viz. $3244 \text{ £ } 4 \text{ s.}$ is here found to be the present Worth,
which is less than that found by the Pen, by 15 . being
but very inconsiderable..

Example 3. In Quarterly Rents.

What is 80 l. per Annum , to continue $31 \text{ Years and a Half}$, Worth in present Money, at the Rate of $7 \text{ per Cent. per Annum}$; If the Rents be paid Quarterly, (*viz.* $20 \text{ l. every Quarter of a Year}$?)

In this Question $U = 20$, $t = 31,5$ and $R = 1,07$
To find P , As before.

First $R = 1,07$ its Log. is $0,029384$ } Multiply
And $t = 31,5$ }

The Product is the Logarith. $0,925596$ of $8,4255$
And $8,4255 - 1 = 7,4255$ and $7,4255 \times 20 = 148,51$
for the Dividend.

Again

Again $R=1,07$ the $\frac{1}{4}$ of its Log. is $0,007346$ whole Number is $1,01706$ and $1,01706 - 1 = 0,01706$

Next $8,4255 \times 0,01706 = 0,143739$ the Divisor.

Then $0,143739 \div 148,510000 = 1033,185 = P$.

That is 1033 l. 3 s. $8\frac{1}{2}$ d. will be the present Worth as was required.

Or having found the Multiplier $0,01706$ As above; Then the rest of the Work may be perform'd by *Logarithms*.

Thus, $R=1,07$ its Log. is $0,029384$ } Multiply
And $t = 31,5$

Again $U=20$ its Log. is $0,925596$ } Add
 $1,301030$

The Sum is the Logarith. $2,226626$ of $168,51$

From this $168,51$ Subtract the Quarterly Rent 20 ,
Remains $148,51$ its Log. is $2,171756$.

Log. of R Multiplied with t is $0,925596$ } Add
The Multiplier $0,01706$ its Log. $8,231979$

From 1st Log. Subst. this Log. $9,157575$

There Remains the Logarith. $3,014181$ of $1033,18$
Viz. 1033 l. 3 s. 7 d. is the present Worth found by the *Logarithms*.

Case 2. When P , t , and R , are given; To find U
That is, To find what Annuity, or Lease, &c. may be Purchased for any proposed Sum; to continue any assign'd Time, at any given Rate of Interest.

Rule.

First Multiply the Logarithm of R with the Time, and find the Number which belongs to the Product, and call that Number x , (as in the last Section.)

Again, Multiply the same Log. of R with the Time more 1 , and find the Number which belongs to that Product. Then Multiply the Difference of those Two

Numbers with the proposed present Worth, and their Product will be a Dividend.

And the first Number called x , being made Less by 1, will be the Divisor, by which if the aforesaid Dividend be Divided, the Quotient will shew the Annuity requir'd for Yearly Payments.

Example 1. For Yearly Rents.

What Annuity, or Yearly Rent, to continue Twenty one Years, may be Purchased for 3205 l. 5 s. at the Rate of Five per Cent. per Annum?

Here is given $P = 3205,25$ $t = 21$ and $R = 1,05$
To find U .

First $R = 1,05$ its Log. is 0.021189 } Multiply
And $t = 21$

The Product is the Log. 0.444969 of 2,7859 call'd x

Again $R = 1,05$ its Log. is 0.021189 } Multiply
And $t + 1 = 22$

Their Product is the Logar. 0.466158 of 2,9252
Then $2,9252 - 2,7859 = 0,1393$ the Multiplier.

Next $P = 3205,25 \times 0,1393$ gives 446,4913 for the Dividend.

And the Number called x 2,7859 $- 1 = 1,7859$ is the Divisor.

Then $1,7859 \mid 446,4913$ (250 l. = U the Annuity requir'd, for Yearly Payments.

Or having once found the Two first Numbers, and by them form'd the Multiplier, and Divisor, as above, the Multiplication, and Division may be perform'd by *Logarithms*.

Thus $P = 3205,25$ its Log. 3.505861 } Add
Multipl. 0,1393 its Log. 9.143951

Sum 2.649812 } Subtract
The Divis. 1,7859 its Log. 0.251857

Remains the Logarithm 2.397955 of 250 = U
Viz. 250 l. will be the Yearly Annuity, as before:
But this Case is more Universally Resolved by the following Rule.

Rule

First Multiply the Log. of R the Amount of 1 l. with the Time, and find the Number which belongs to that Product, which call x , (As before.)

Rule 2. Next, Multiply the proposed present Worth with $R - 1$, viz. with the Ratio of the given Rate, and Multiply their Product with the first Number, called x .

Then Divide the last Product by the Number called x made Less by 1, and the Quotient will shew the Annuity, or Rent for Yearly Payments, (As before.)

As for Instance in the last Example, wherein $P = 3205,25$ $t = 21$, and $R = 1,05$ To find U .
 Then $R = 1,05$ its Log. $0,02189$ } Multiply
 And $t = 21$ }

The Product is the Log. $0,444969$ of $2,7859$ call'd x .
 Next $R - 1 = 0,05$ And $P = 3205,25 \times 0,05 = 160,2625$
 Again, $160,2625 \times 2,7859 = 446,753$ the Dividend.
 And $2,7859 - 1 = 1,7859$ for the Divisor.

Then $1,7859 \overline{) 446,753}$ ($250 = U = 250$ l. the Annuity, or Rent for Yearly Payments.

The same may be perform'd by Logarithms.

Thus, If $R = 1,05$ and $t = 21$; Then $2,7859$ the Number called x will be found as before.

And if $P = 3205,25$ its Log. is $3,505861$ }
 $R - 1 = 0,05$ its Log. is $8,698970$ } Add
 Num. call'd x $2,7859$ its Log. is $0,444969$ }
 Sum $2,649800$ } Subst.
 Num. $x - 1 = 1,7859$ its Log. $0,251857$ }

There Remains the Logarithm $2,397943$ of 250
 That is, 250 l. will be the Rent for Yearly Payments, (As before.)

But if the Annuity, or Rent, &c. is to be paid either Half-yearly, or Quarterly, &c. Then

Then, *Instead of Multiplying the proposed present Worth of the Annuity, with $R - 1$ (viz. with the Ratio of the Rate, as in Rule 2.) you must take such a Part of the Log. of R , as the Time requires (viz. the $\frac{1}{2}$, the $\frac{1}{4}$ or $\frac{1}{12}$, &c.) and its Number made less by 1, must be the Multiplier; and then proceed by Rule 2. As in the following Examples.*

Example 2. In Half-yearly Payments.

What Annuity, or Rent, to be paid every Half-Year, and to continue 21 Years, may be Purchased for the Sum of 3244 l. 5 s. at the Rate of 5 per Cent. per Annum?

In this Question there is given $P = 3244,25$ $t = 21$ and $R = 1,05$ To find U , the Half-yearly Rent.

Thus $R = 1,05$ its Log. $0,021189$ } Multiply
And $t = 21$ }

The Product is the Log. $0,444969$ of $2,7859$ call'd x :

Then $2,7859 - 1 = 1,7859$ the Divisor. So far the Work is the same as for Yearly Payments.

Next $R = 1,05$ the $\frac{1}{2}$ of its Log. is $0,010594$ whose Number is $1,0247$, and $1,0247 - 1 = 0,0247$ the Multiplier instead of $0,05 = R - 1$.

Then $P = 3244,25 \times 0,0247$ the Product is $80,133$
And $80,133 \times 2,7859$ the Number call'd x , the Product is $223,2432$ for the Dividend,

Then $x - 1 = 1,7859$) $223,2432$ ($125 = U$.

That is 125 L. will be the Half-yearly Annuity, or Rent as was requir'd by the Question.

Or having once found the Number call'd x , and taken 1 from it for the Divisor; And the Multiplier used instead of $R - 1$, As before: Then the Multiplication, and Division may be perform'd by Logarithms; Thus.

$R =$

$R = 1,05$ its Log. \times with t is 0.444969 }
 And $P = 3244,25$ its Log. is 3.511114 } Add
 Multiplier $0,0247$ its Log. is 8.392697 }
 Sum 2.348780 }
 The Divif. is $1,7859$ its Log. is 0.251857 } Subtract
 There Remains the Logarith. 2.096923 of $125 = U$
 &c.

Example 3. In Quarterly Payments.

*What Annuity, or Rent, to be paid every Quarter of a Year, and to continue 31 Years and a Half, may be Purchased for 1033*l.* 3*s.* 8*d.* at the Rate of 7 per Cent. per Annum?*

Here is given $P = 1033,185$ $t = 31,5$ and $R = 1,07$
 To find U , the Quarterly Payments.

First $R = 1,07$ its Log. is 0.029384 } Multiply
 And $t = 31,5$ }

The Product is the Log. 0.925596 of $8,4255$ call'd x
 And $8,4255 - 1 = 7,4255$ for the Divisor.

Again, $R = 1,07$ the $\frac{1}{4}$ of its Log. is 0.007346 whose Number is $1,01706$ and $1,01706 - 1 = 0,01706$ for the Multiplier, instead of $0,05 = R - 1$.

Then $P = 1033,185 \times 0,01706$ the Product is $17,6261$
 And $17,6261 \times 8,4255$ the Number call'd x , the Product will be $148,5087$ the Dividend.

Then $7,4255 \mid 148,5087$ ($20 = U$; That is 20 *l.* will be the Quarterly Annuity, or Rent as was requir'd.

The same may be found by *Logarithms*, as in the last Example of Half-yearly Rents; which I suppose is needless to be set down.

Case 3. When U , P , and R are given; To find t .
 That is, When any Annuity, with its proposed Value or Purchase, and the Rate of Interest *per Cent.* &c. are given; Thence to find how long the Purchaser ought to enjoy it. Rule.

Rule. *First, Multiply the proposed Value or present Worth of the Annuity, with $R - 1$. (Viz. with the Ratio of the given Rate of Interest) and Subtract the Product from the Annuity, the Remainder will be the Divisor.*
Next, Divide the Annuity by that Divisor, and find the Log. of the Quotient; Then if that Log. be Divided by the Log. of R the Amount of 1 l. the Quotient will shew the Time, for Yearly Payments.

Example 1. For Yearly Rents.

Suppose 3205 l. 5 s. were Lent upon an Annuity of 250 l. per Annum; In what Time would it pay off that Debt, allowing the Creditor 5 per Cent. per Ann.?

In this Question there is given, $P = 3205,25$
 $U = 250$. and $R = 1,05$ To find t .

First $P = 3205,25$ } Multiply
 $R - 1 = 0,05$

And $U = 250 - 160,2625$ leaves 89,7375 the Divisor.

Next, 89,7375) 250,0000 (2,7859 whose Logarithm is 0.444969.

Then $R = 1,05$ its Log. is 0.021189) 0.444969. (21 = t .
 That is 21 Years is the Time required in the Question.

The same may be perform'd by Logarithms; Thus,
 If $P = 3205,25$ its Log. is 3.505863 } Add
 And $R - 1 = 0,05$ its Log. 8.698970 }

The Sum is the Logarithm. 2.204833 of 160,2625

Next, If $U = 250$ its Log. is 2.397940 } Subtract
 $250 - 160,2625 = 89,7375$ Log. 1.952972 }
 $R = 1,05$ its Log. is 0.021189) 0.444968 (21 = t ,
 Viz. 21 Years is the Time, if the Payments are Yearly.

But

But if the Annuity, or Rent is to be paid, either by Half-yearly, or Quarterly Payments, &c.

Then { Instead of Multiplying the proposed present Worth of the Annuity with $R - 1$ (as before,) you must take such a part of the Logar. of R the Amount of 1 l. as the Time requires, and the Number answering to that part, being made Less by 1, will be the Multiplier; and then you may proceed on as before, according to the Rule.

Example 2. For Half-yearly Rents.

Admit a Man should give 3244 l. 5 s. for an Annuity, or clear Rent of 250 l. per Annum; to be paid Half-yearly (viz. 125 l. every Half-Year;) How long must he Enjoy that Annuity, to be allow'd Five per Cent. &c.?

Here is given $U = 125$, $P = 3244,25$ and $R = 1,05$
To find t , the Time or Number of Years

First $R = 1,05$ the $\frac{1}{2}$ of its Log. is 0.010594 whose Number is 1,0247 and $1,0247 - 1 = 0,0247$ the Multiplier instead of $0,05 = R - 1$.

Then $P = 3244,25 \times 0,0247 = 80,1329$.

And $U = 125 - 80,1329$ leaves 44,8671 the Divis.

Next 44,8671) 125,0000 (2,786 whose Logarithm is 0,444981 for the Dividend.

Then $R = 1,05$ its Log. is 0.021189) 0.444981 (21 = t .

That is, 21 Years is the Time that is required in the Question.

The same perform'd by Logarithms.

Suppose $R = 1,05$ then the $\frac{1}{2}$ of its Log. 0.010594 whose Number is 1,0247 and $1,0247 - 1 = 0,0247$ the Multiplier instead of $R - 1$; As before.

Then $P = 3244,25$ its Log. is 3.511114 } Add
The Multipli. 0,0247 its Log. 8.392697 }

Their Sum is the Logarithm 1.903811 of 80,133
And

Next $U = 129$ its Logar. is 2.096910 } Subf.
 And $125 - 80,133 = 44,867$ its Log. 1.651927 }
 Then $R = 1,05$ its Log. is 0.021189) 0.444983 ($21 =$
Viz. 21 Years, &c. As before.

Example 3. In Quarterly Payments.

Suppose one should give 1033 l. 3 s. 8 d. for an Annuity of 80 l. per Annum, to be paid Quarterly (viz. to have 20 l. every Quarter of a Year;) How Long may be Enjoy it, to be allowed Seven per Cent. per Annum ?

In this Question there is given $U = 20$, $P = 1033,185$ and $R = 1,07$ To find t , the Time or Num. of Years.
 Thus, $R = 1,07$ the $\frac{1}{4}$ of its Log. is 0.007346 whose Number is $1,01706$ and $1,01706 - 1 = 0,01706$ for the Multiplier, to be used instead of $0,07 = R - 1$.
 Then $P = 1033,185 \times 0,01706 = 17,6261$; which being taken from $U = 20$, Leaves $2,3739$ the Divisor.
 Then $2,3739$ ($20,0000$ ($8,425$ its Log. 0.925570 .
 Lastly $R = 1,07$ its Log. 0.029384) 0.925570 ($31,5 =$
Viz. the Time requir'd will be 31 Years and a Half.

Which may also be found by *Logarithms*.

Thus, If $R = 1,07$ the $\frac{1}{4}$ of its Log is 0.007346 whose Number is $1,01706$ and $1,01706 - 1$ Leaves $0,01706$ for the Multiplier used instead of $R - 1$.

Then $P = 1033,185$ its Log. is 3.014181 } Add
 Multiplier $0,01706$ its Log. 8.231979 }

Their Sum is the Logarithm 1.246160 of 176262
 And if $U = 20$ then $20 - 17,6262 = 2,3738$ the Divis.

Again $U = 20$ its Logar. is 1.301030 } Subtra ϕ
 Divisor $2,3738$ its Log. is 0.375444 }

$R = 1,07$ its Log. is 0.029384) 0.925586 ($31,5 = t$, &c.

Case

Case 4. By having U , t , and P , given; To find R ; That is, When any Annuity, or Rent *per Annum*, with the Time of its Continuance, and its proposed present Worth for that Time are given; Thence to find what Rate of Interest *per Cent.* &c. is allow'd the Purchaser.

This Case is so difficult, that the Ingenious *Michael Dary*, in his Treatise of *Interest* Epitomiz'd, *Pag. 7.* sets it down as very troublesome to be Resolved, in regard (*says he*) that R (*the thing sought*) cannot be brought to one Side of the Equation solely, and the other Side clear. But I rather think that the true Cause which rendred a perfect Solution of this Case so troublesome to him (*or indeed impossible at that time*) was because the Resolving of such high Equations as this Case requires, was not in his Time so well known as now it is.

For the Equation, by which the true Value of R may be found, is this,

$$\text{Vir. } \frac{U}{P} = \frac{U}{P} R^t + R^t - R^{t-1} - 1$$

Which I Reduce to the following Equation, as being much better for our present Purpose,

$$\text{Vir. } U = UR^t - PR^t \times R - 1.$$

Either of these Equations is more Adfected than that in *Page 67*; and require an Algebraical Solution to discover the true Value of R ; And can be no otherwise found here, but only by the same manner of Approachment, as before in the 4th Case of the last Section: However, in order to render that Approximation as easie and short as possibly I can, observe the following Directions.

Vir.

First, Assume such a Number for the Value of R , as you guess may be the Amount of 1 l. at the Rate of the

the Interest required, (as before in Page 68, &c.) Then Multiply the Log. of that supposed R with the Time, and find the Number which belongs to that Product, and Multiply it with the Annuity; Call the Product z .

Next, Multiply the same Number with the proposed present Worth, and Multiply their Product with $R - 1$, (viz. with the Ratio of the supposed Rate:)

Then Subtract the last Product from the Product call'd z . If the Remainder be Equal to (viz. be the same with) the Annuity, you have hit right upon the true Value of R : But if it be Less than the Annuity, then is the Number of R taken too Big: And on the contrary, if it be more than the Annuity, that R is too Little; and another Tryal must be made accordingly.

Example 1. In Yearly Rents.

Suppose there were paid 3205 l. 5 s. for an Annuity, or a Lease of 250 l. per Annum, to continue 21 Years; What Rate of Interest per Cent. &c. is allow'd to the Purchaser?

In this Question there is given, $U = 250$, $t = 21$, and $P = 3205,25$ To find R the Amount of 1 l.

First, I will suppose R to be 1,06 That is the Amount of 1 l. at the Rate of 6 per Cent.

Then if $R = 1,06$ its Log. is 0,025306 } Multiply
And $t = 21$ }

The Product is the Logarith. 0,531426 of 3,3996

Then $U = 250 \times 3,3996$ the Product is 849,9 which is the Number call'd z .

Next $P = 3205,25 \times 3,3996$ the Product is 10896,5679 And $10896,5679 \times 0,06$ (viz. with $R - 1$) the Product will be 653,7941 Then if this Number be Subtracted from 849,9 the Number call'd z , there will Remain 196,1059 which is Less than 250 the Annuity: And therefore I conclude that the supposed $R = 1,06$ is too Big.

Again,

Again, For a Second Tryal, I will suppose $R = 1,05$
 Then if $R = 1,05$ its Log. is 0.021189 } Multiply
 And $t = 21$

The Product will be the Log. 0.444969 of 2.7859
 And $250 \times 2.7859 = 696,475$ the Number called γ .

Next $3205,25 \times 2.7859 = 8929,5$ which being Multiplied with $0,05 = R - 1$, the Product is $446,475$
 Then γ , $696,475 - 446,475 = 250,000$ which is the very same with the Annuity : Whence I conclude that the true Value of R is $1,05$ And $R - 1 = 0,05$ will be the Ratio of the Rate of Interest sought.

Then it will be, As $1 : 15$ to $0,05 ::$ So is $100 : To 5$ the Rate of Interest *per Cent.* as was required for Yearly Payments.

Now these Trials for finding the true Value of R , may be perform'd by *Logarithms* with a very little trouble. As for Instance in this last Example,

Wherein $U = 250$, $P = 3205,25$ and $t = 21$,
 To find R .

Suppose $R = 1,06$ its Log. is 0.025306 } Multiply
 And $t = 21$

Sum 0.531426 } Add
 Next $U = 250$, its Log. is 2.397940

The Sum is the Logarith. 2.929366 of $849,9$ the Number called γ .

Again $R = 1,06$ its Log. \times with t , is 0.531426 }
 And $P = 3205,25$ its Log. is 3.505863 } Add
 $R - 1 = 0,06$ its Log. is 8.778151

The Sum is the Logarith. 2.815440 of $653,79$
 Then γ , $849,9 - 653,79 = 196,11$ which is Less than 250 the Annuity : Whence it appears that $R = 1,06$ is too Big.

2ly. Let $R=1,05$ its Log. 0.021189 } Multiply
 And $t=21$ }

Sum 0.444969 } Add
 Next, $U=250$ its Log. 2.397940 }

The Sum is the Logar. 2.842909 of $696,48$ call'd z .

Again $P=3205,25$ its Log. is 3.505863 }
 $R=1,05$ its Log. \times with t , is 0.444969 } Add
 And $R-1=0,05$ its Log. is 8.698970 }

Their Sum is the Logarithm 2.649802 of $446,48$

Then $z, 696,48 - 446,48 = 250$, which is just the same with the Annuity; Therefore the true Value of R is $1,05$ &c. As before.

But if the Annuity, or Rent is to be paid, either Half-yearly, or Quarterly, &c.

Then { *Instead of Multiplying with $R-1$, viz. with the Ratio of the supposed Rate of Interest. (as above) you must take such a part of the Log. of R , (the assumed Amount of 1 l.) as the Time requires, and make its Number the Multiplier, &c. As in the following Examples.*

Example 2. In Half-yearly Payments.

Admit 3244 l. 5 s. were given for an Annuity, or Lease of 250 l. per Annum, to be paid Half-yearly, (viz. 125 l. every Half-year) and to continue 21 Years; What Rate of Interest per Cent. is allow'd the Purchaser.

In this Example there is given $U=125$, $t=21$, and $P=3244,25$ To find R the Amount of 1 l.

First, I suppose $R=1,06$ its Log. 0.025306 } Multiply
 And $t=21$ }

Their Product is the Logarith, 0.531426 of $3,3996$

Then $U=125 \times 3,3996$ the Product is $424,95$ call'd z .

Next $P=3244,25 \times 3,3996$ the Prod. is $11029,1523$

Again,

Again, If $R=1,06$ the $\frac{1}{2}$ of its Log. is $0,012653$ whose Num. is $1,02956$ and $1,02956 - 1 = 0,02956$ the Multiplier instead of $0,06 = R - 1$.

Then $11029,1523 \times 0,02956 = 326,0217$ which being Subtracted from $424,95$ called r ; there will Remain $98,9283$ which is Less than 125 the Half-yearly Annuity; Whence I conclude that the supposed $R=1,06$ is taken too Big.

And therefore for a second Tryal I will suppose $R=1,05$ and proceed by *Logarithms* only:

Then if $R=1,05$ its Log. $0,021189$ } Multiply
And $t=21$

Sum $0,444969$ } Add
Next $U=125$ its Log. is $2,096910$

Their Sum is the Logar. $2,541879$ of $348,24$ call'd r .

Again, if $R=1,05$ the $\frac{1}{2}$ of its Log. is $0,010594$ whose Num. is $1,0247 - 1$, Leaves $0,0247$ the Multiplier.

Then $P=3244,25$ its Log. is $3,511114$ } Add
And $R=1,05$ its Log. \times with t , is $0,444969$ }
Multiplier $0,0247$ its Log. is $8,392697$ }

Their Sum is the Logarith. $2,348780$ of $223,24$

Then $r, 348,24 - 223,24 = 125$ which is the very same with 125 the Half-yearly Annuity; And therefore I conclude that $R=1,05$ and $R-1=0,05$ is the Ratio of the Rate of Interest sought.

Consequently, As $1 : 15$ to $0,05 ::$ So is $100 : To 5$ the true Rate of Interest *per Cent.* &c.

I shall close up this Case with an *Example* of finding the Rate of Interest, that is allow'd to the Purchasers of the *Parliamentary* Annuities; and Resolve it by *Logarithms* only.

Example 3. Of Quarterly Payments.

There is 1600 l. paid for an Annuity of 100 l. per Annum, to continue 99 Years, and to be paid Quarterly,

viz. 25 l. every Quarter of a Year during that Time:
*'Tis required to find what Rate of Interest per Cent. is
 allow'd the Purchaser.*

In this Question there is given $U = 25$, $t = 99$,
 and $P = 1600$; To find R , &c.

First, I will here suppose the Rate of Interest sought
 to be 6 per Cent. for the Reasons mentioned in *Pag. 49.*

Then if $R = 1,06$ its Log. 0.025306 } Multiply
 And $t = 99$ }

The Product is 2.505294 } Add.
 Next, $U = 25$ its Log. 1.397940 }

The Sum is the Log. 3.903234 of 8002,6 call'd γ .

Again, if $R = 1,06$ the $\frac{1}{4}$ of its Log. is 0.006326
 whole Number is 1,01467 Less 1, Leaves 0.01467
 the Multiplier.

Then $P = 1600$ its Log. is 3.204120 }
 The Log. of R Multipl. with t 2.505294 } Add
 Multiplier 0.01467 its Log. 8.166430 }

Their Sum is the Logarith. 3.875844 of 7513,5
 which being taken from 8002,6 = γ Leaves 489,1
 which is More than 25 = U ; And therefore the Value
 of R is taken too Little.

And for a Second Tryal I do Assume $R = 1,064$

Then if $R = 1,064$ its Log. 0.026942 } Multiply
 And $t = 99$ }

The Product is 2.667258 } Add
 Next $U = 25$ its Log. is 1.397940 }

The Sum is the Logarith. 4.065198 of 11619,8 = γ .

Again, the $\frac{1}{4}$ of the Log. of R is 0.006735 its
 Num. is 1,01563 And 1,01563 - 1 = 0.01563 the
 Multiplier.

Then

Then $P=1600$ its Log. 3.204120 }
 Log. of $R \times$ with t , is 2.667258 } Add
Multiplic. 0.01563 its Log. 8.193959 }

Their Sum is the Logar. 4.065337 of 11623,5 which is More than 11619,8 = t ; And therefore the Value of this $R=1,064$ is some small matter too Big; Consequently R is some Number between 1,06 and 1,064, and is but a very little Less than 1,064

Let $R=1,0638$ its Log. 0.026860 }
 And $t=99$ } Multiply

Product 2.659140 }
 Next $U=25$ its Log. is 1.397940 } Add

The Sum is the Logar. 4.057080 of 11414,6 = t .
 Again $R=1,0638$ the $\frac{1}{4}$ of its Log. is 0,006715 whose Num. is 1,01558 Less 1 is 0,01558 the *Multiplicator*.

Then $P=1600$ its Log. is 3.204120 }
 Log. of R Multiplied with t 2.659140 } Add
Multiplicat. 0,01558 its Log. 8.192567 }

The Sum is the Logarithm 4.055827 of 1137,7

Then the Num. call'd t , viz. 11414,6 — 11377 = 37,6 which is now a small matter More than 25 = U .

Consequently $R=1,0638$ is a very small matter too Little, which I judge must be betwixt 1,0638 and 1,0639; And accordingly I make one other Tryal with assuming the Value of $R=1,063855$ and find it pretty near to the Truth.

Consequently $R-1=0,063855$ the Ratio of the Rate. And then it will be As 1 : 0,063855 :: 100 : 6,3855 That is, 6 l. 7 s. 8 $\frac{1}{2}$ d. will be very near the true Rate of Interest *per Cent.* As was required.

And for a Confirmation of the Truth thereof, let us take it for granted that $R=1,063855$ As above; And let there be given the Quarterly Annuity, or $U=25$ l. the Time or $t=99$ Years; To find P the present Worth (*per Case 1.*) which we already know

know must be 1600 *l.* if the Value of *R* be truly found. Then,

If $R = 1,063855$ its Log. is 0.026882 } Multiply
And $t = 99$

Their Product is 2.661318 } Add
Next $U = 25$ its Log. is 1.397940 }

The Sum is the Logarithm 4.059258 of 11461.94
From this 11461.94 Subtract the Quarterly Rent 25,
there Remains 11436.94 the Dividend.

Again $R = 1,063855$ the $\frac{1}{4}$ of its Log. is 0.006720
whose Num. is 1,015592 and $1,015592 - 1 = 0.015592$
the Multiplier instead of $0,063855 = R - 1$.

The Dividend 11436.94 its Log. is 4.058312

Log. of *R* Multiplied with t , is 2.661318 } Add
Multiplier 0,015592 its Log. is 8.192902 }

From the upper Log. Subt. this Log. 0.854220 Sum

There Remains the Logarithm 3.204092 of
(1599,9 = *P*.)

That is, the present Worth thus found is 1599 *l.* 18 *s.*
which wants but 2 *s.* of 1600 *l.* the true present Worth:
Whence we may be assured, that the Value of *R*, and
consequently the Rate of Interest *per Cent.* found as
above, is very near the Truth, for Quarterly Pay-
ments. But supposing those Annuities were to be paid
but once a Year, then the Rate of Interest will be only
6 *l.* 5 *s.* *per Cent.* or very near it, as may be easily
Tried, either by finding the present Worth, as in
Example 1. Page 74. Or by finding the Annuity, as
in *Example 1. Page 78, &c.*

Thus you have a full Solution at large to the Eight
Cases, which include all the Varieties that can be pro-
posed, either relating to the Arrears; or about the
Purchasing of Annuities, and the Taking of such
Leases as are either in present Possession when the Con-
tract is made, Or are to be immediately Entered
upon.

upon. But for such Questions, as relate to Annuities, and Taking such Leases, &c. as are in Reversion, they require a further Consideration.

Sect. 4. To find the Present Worth of Annuities, or Leases, &c. in Reversion at Compound Interest.

I have already touch'd upon this Part of the Work Pag. 47, &c. as it may be performed at *Simple Interest*; But shall here handle it more fully in all its useful Cases. For altho' they only depend upon the due Application of the Rules already laid down; yet it will be convenient to shew how those Rules are to be truly Applied, according as the particular Cases require.

Case 1. When the Annuity, or Yearly Rent, with the Time of its Continuance in Possession, and the Time of the Reversion, (*viz. the Time it's not to be in Possession*) are given; To find its Present Worth, at any assign'd Rate of Interest per Cent.

The Solution of this Case requires Two distinct Operations, which must be thus perform'd.

First { Find the Value or Present Worth of the proposed Annuity, or Rent for the given Time of its Continuance, as if it were immediately to be Entered upon.

Then { Find what Principal or Sum, being forborn at Interest, during the Time of the Reversion, would Amount to or Raise the aforesaid Value; and that Principal will be the Present Worth of the proposed Annuity, &c. in Reversion.

Example in Yearly Rents.

There is the Reversion of a Lease, of 175 l. per Ann. to be Lett for Eleven Years, which are not to commence until after Nine Years are expired: 'Tis required to find the

the Present Worth of that Lease, allowing the Rate of Six per Cent. Interest to the Purchaser.

Or thus :

There is One has Nine Years to come in a Lease of 175 l. per Annum, and he's desirous to enlarge his Time Eleven Years more (viz. to Enjoy his Lease Twenty Years yet to come;) What Sum must he give in ready Money for that Purchase, to be allow'd the Rate of 6 per Cent. &c.?

In the first Work of this Example there is given $U = 175$, $t = 11$, and $R = 1,06$ To find P the Present Worth, As in Case 1. Page 74.

Thus $R = 1,06$ its Log. is 0.025306
And $t = 11$ } Multiply

The Product is 0.278366
Again $U = 175$ its Log. is 2.243038 } Add

The Sum is the Logarithm 2.521404 of $332,203$
Then from $332,203$ Subtract the Rent 175 , and there Remains $157,203$ its Log. is 2.196459

The 1st Log. \times with $t = 11$ is 0.278366
And $R = 1,06$ its Log. is 8.778151 } Add

From up. Log. Subst. this Log. 9.056517

There Remains the Logarith. 3.139942 of $1380,2 = P$ the Present Worth, supposing the Rent were to be immediately Entered upon; but because it is not so, therefore, for the Second Part of the Work there is given $A = 1380,2$, $t = 9$, and $R = 1,06$ To find P the Principal, As in Case 2. Page 54.

Thus $A = 1380,2$ its Log. is 3.139943
 $R = 1,06$ its Log. $\times 9 = t$, is 0.227754 } Subtract

And there Remains the Log. 2.912189 of $816,937 = P$
Viz. 816 l. 18 s. 9 d. will be the true Present Worth of that Lease in Reversion, as was required, viz. for Yearly Rents.

But

But if the Rents are to be paid, either *Half-yearly*, or *Quarterly*, &c. Then the first Part of the Work must be found as in their respective Examples, viz. For Half-yearly Payments it must be found as in the Example, Page 75. And for Quarterly Payments as in the Example, Page 76. And then proceed on as above.

Case 2. To find what Annuity, or Yearly Rent in Reversion, may be purchased for any proposed Sum, at any assign'd Rate of Interest *per Cent.* When the Time the Annuity is not to be Enter'd upon; And the Time of its Continuance are both given.

The Solution of this Case is but the Reverse of the Last, and doth also require Two distinct Operations.

First { Find what the Sum proposed to be laid out in the Reversion, would Amount to, supposing it were forborn at the given Rate of Interest, during the Time that the Annuity is not to be in Possession.

Then { Find what Annuity, or Yearly Rent that Amount will Purchase, and that will be the Answer requir'd.

Example in Yearly Rents.

What Annuity, or Yearly Rent, to commence Nine Years hence, and then to continue Eleven Years after, may be Purchased for 816 L. 18 s. 9 d. ready Money; at the Rate of 6 per Cent. &c.

In the first Work of this Question there is given, $P = 816,937$ $R = 1,06$ and $t = 9$ (the Time that the Annuity is not to be Enter'd upon;) To find A ; As in Case 1. Page 53.

Thus $R = 1,06$ its Log. 0.025306 } Multiply
And $t = 9$

The Product is 0.227754 } Add:
Next $P = 816,937$ its Log. 2.912189

The Sum is the Logar. 3.139943 of $1380,2 = A$ the Amount

Amount of 816 l. 18 s. 9 d. which we must now call P for the second Part of the Work ; wherein there is given $P = 1380,2$ $R = 1,06$ and $t = 11$ (*the Time that the Annuity is to be Enjoy'd*) To find U , As in *Case 2. Page 77.*

Thus $R = 1,06$ its Log. is 0.025306 } Multiply
And $t = 11$ }

The Product is the Log. 0.278366 of $1,8983$
And $1,8983 - 1 = 0,8983$ is the Divisor.

Again $R = 1,06$ its Log. is 0.025306 } Multiply
And $t + 1 = 12$ }

Their Product is the Logar. 0.303672 of $2,0122$
And $2,0122 - 1,8983 = 0.1139$ the Multiplier.

Then $P = 1380,2$ its Log. is 3.139943 } Add
Multiplier $0,1139$ its Log. 9.056524 }

Sum 2.196467 } Subtract
The Divisor $0,8983$ its Log. 9.953421 }

There Remains the Logar. 2.243046 of $175 = U$.
That is, 175 l. *per Annum* is the Rent, or Annuity required by the Question.

But if the Rents are to be paid every *Half-Year*, or every *Quarter*, &c. Then the second Part of the Work must be perform'd as in their respective Examples ; *Viz.* For Half-yearly Payments, U must be found, as in the Example, *Page 80.* And for Quarterly Payments as in the Example, *Page 81, &c.*

Case 3. Any Annuity, or Yearly Rent, and the Sum that's propos'd to be paid for a Lease of it, to commence after a certain Term or Number of Years are past, being given ; Thence to find how Long that Lease ought to continue in the Possession of the Purchaser, allowing him any assign'd Rate of Interest *per Cent.*

This Case does also require Two separate Operations.
Viz.

Viz. { First, To find what Sum the proposed Money to be laid out in the Reversion, would Amount to; &c. As in the first part of the Last Case.

Then { Having the Annuity, with its present Worth or Value, and the Rate of Interest given; To find the Time, &c. As in Case 3. Page 82.

Example in Yearly Rents.

Suppose a Debt of 816 l. 18 s. 9 d. were proposed to be paid off, by making over a Lease of 175 l. per Annum in Reversion, that is not to be Enter'd upon, by the Creditor, until Nine Years are past; The Question is, How many Years the Creditor may Enjoy that Lease, to have his Debt Cleared; and to be allowed the Rate of 6 per Cent. per Annum.

For the first Work in this Question there is given $P = 816,937$ $R = 1,06$ and $t = 9$ (the Time before the Lease is to commence,) To find A , which will be found $= 1380,2$ as in the first Part of the Last Case; which we must here also call P , for the Second Part of the Work.

Then there will be given new $P = 1380,2$ $U = 175$ and $R = 1,06$ To find the $t =$ to the Time which the Lease is to be Enjoy'd; As in Example 1. Pag. 82.

Thus $P = 1380,2$ its Log. is 3.139942 } Add
 $R - 1 = 0,06$ its Log. is 8.778151 }

The Sum is the Logarithm 1.918093 of 82,812

Next $U = 175$ its Log. is 2.243038 } Substra^d
 $175 - 82,812 = 92,188$ its Log. 1.964674 }

$R = 1,06$ its Log. is 0.025306) 0.278364 (11 = t .
Viz. 11 Years will be the Time required by the Question; Supposing the Rents to be paid but once a Year.

But if the Rents are to be paid every Half-Year, Then t , the Time the Lease is to be Enjoy'd, may
 K be

98 Of Compound Interest, &c.

be found as in *Example Page 82*. And if Quarterly, then as in *Example Page 84, &c.*

Now these Three Cases include all the usual (and useful) Questions that relate to Purchasing Annuities, or Taking of Leases in Reversion; And altho' I've made choice of shewing how to Resolve each of them by Two separate and distinct Operations, (being, as I presume, the easiest Way to be understood) yet they may be Resolved at one, by a due Reduction of the following Equation. For

Putting $\begin{cases} t = \text{the Time the Lease, \&c. is to be Enjoy'd;} \\ T = \text{the Time of its Reversion;} \end{cases}$ Then

$$\text{it will be } UR^t - U = PR^T \times R^t \times R - 1.$$

And from hence the Value of R , viz. the Rate of Interest may also be found; which I take to be a Work more of Curiosity, than of any great Use: For the Rate of Interest is always supposed to be known, it being one of the main or chief Considerations in all Bargains about Taking, or Letting of Leases, &c. And therefore the finding of R in the Case of Reversions, (or indeed in the Two 4th Cases of the Two last Sections) is rather for a Proof of the Work, and to Compleat the Rounds, than for any thing else. However, if any one will (out of his Curiosity) give himself the Trouble of doing it, he may proceed by such Approximation as in the aforesaid Sections, having a due regard to the different Values of t , and T . But for Brevity sake I shall omit giving Examples thereof.

C H A P.

C H A P. V.

Of Purchasing Free-hold or Real Estates,
and how to Estimate the Value of
Annuities, and Leases for Lives, &c.

ALL Free-hold or Real Estates, are supposed to be Purchased (or Bought) to continue for Ever; *Vir.* without the Consideration of any limited Time: Therefore the Business of Computing the true Value or Worth of such Estates (*according to Art*) is grounded upon a Rank, or Series, of Geometrical Proportionals, continually Decreasing *ad Infinitum*; (*As may be seen in the aforesaid Young Mathematician's Guide, Pag. 276.*) from whence the following Rules are deduced.

Sect. 1. Of Purchasing Free-hold Estates.

I shall here make use of the same Letters to denote the several Parts of the Question, as in the last Chapter, save only that: the Time to limit the Number of Payments, is not concern'd.

Vir. $\left\{ \begin{array}{l} U = \text{The proposed Rent of an Estate.} \\ P = \text{The Purchase, or Worth of that Estate.} \\ R = \text{The Amount of 1 l. at any Rate of Interest.} \end{array} \right.$

Then it will be $P R - P = U$.

This Equation admits of Three different Cases.

Case 1. The Annual Rent of any Free-hold Estate being known; To find what it's Worth in Ready Money, allowing the Purchaser any assign'd Rate of Interest per Cent.

Rule. $\left\{ \begin{array}{l} \text{Divide the proposed Rent by the Ratio of the} \\ \text{given Rate of Interest (viz. by } R - 1) \text{ and} \\ \text{the Quotient will shew what the Estate is Worth} \\ \text{at that Rate of Interest.} \end{array} \right.$

K 2

Exam-

Example 1.

Suppose a Free-bold Estate of 250 l. Yearly Rent, were to be Sold; What is it Worth, allowing the Buyer the Rate of 6 per Cent. Compound Interest for his Money?

Here is given $U = 250$, $R = 1,06$ To find P .

Thus $R - 1 = 0,06$ $250,00 (4166,6667 = P$.

That is, 4166 l. 13 s. 4 d. is the Present Worth of that Estate; which is 16 Years and 8 Months Value, supposing the Rents to be paid but once a Year.

The same may be found by *Logarithms*,

Thus $U = 250$ its Log. 2.397940 } Subtract
And $R - 1 = 0,06$ its Log. 8.778151 }

There Remains the Log. 3.619789 of 4166,66 = P .
Viz. 4166 l. 13 s. 4 d. &c. As above.

Note, Here it is supposed that the Purchaser is to Enter immediately into Possession of the Estate: But if it be an Estate in Reversion, that is, not to be Entered upon until after some Term of Years are past; Then you must Compute its present Worth, as in Case 1. Pag. 93. And so on in the following Cases, compared with their respective Cases in Sect 4. of the Last Chapter.

If the Rents are to be paid either Half-yearly, or Quarterly, as most generally they are;

Then { Instead of Dividing the proposed Rent by $R - 1$, as above, you must take such a part of the Log. of R the Amount of 1 l. as the Times of Payment requires; and the Number answering to that part, being made Less by 1, must be made the Divisor instead of $R - 1$.

Example in Half-yearly Rents.

Suppose the aforesaid 250 l. per Annum were to be paid by Half-yearly Rents (viz. 125 l. every Half-Year)

What

What would that Estate be then Worth, at the same Rate of Interest, viz. 6 per Cent. &c.?

In this Example there is given $U=125$ and $R=1,06$ To find P . Thus,

First $R=1,06$ the $\frac{1}{2}$ of its Log. is 0.012653 whose Number is $1,029563$ and $1,029563 - 1 = 0,029563$ for the New Divisor instead of $R - 1$.

Then $U=125$ its Log. is 2.096910 } Subtract
Divisor $0,029563$ its Log. 8.470748 }

There Remains the Logar. 3.626162 of $4228,26 = P$.
So that $4228 \text{ l. } 5 \text{ s.}$ is here the present Worth or Value of the same Estate, which is $61 \text{ l. } 11 \text{ s. } 4 \text{ d.}$ More than its Worth for Yearly Rents.

Example 3. For Quarterly Rents.

Let it be required to find what the same Estate (viz. 250 l. per Annum) is Worth, when the Rents are to be paid Quarterly (viz. 62 l. 10 s. every Quarter of a Year) at 6 per Cent. Interest, &c. As before.

Here is given $U=62,5$ $R=1,06$ To find P .

Thus, $R=1,06$ the $\frac{1}{4}$ of its Log. is $0,006326$ whose Num is $1,014674$ and $1,014674 - 1 = 0,014674$ the Divisor instead of $R - 1$.

Then $U=62,5$ its Log. is 1.795880 } Subtract
Divis. $0,014674$ its Log. 8.166548 }

There Remains the Log. 3.629332 of $4259,24 = P$
Viz. $4259 \text{ l. } 4 \text{ s. } 9\frac{1}{2} \text{ d.}$ is now the Value, or present Worth of the same Estate ; which is something above 17 Years Value.

Now from hence it plainly appears, that, although there is no such thing as a limited Time considered in the Purchasing of Free-hold Estates ; yet there is, and ought to be, a due Respect had to the Times of their Rents being paid ; for according to that, their Values are More, or Less Worth.

Case 2. When any Sum of Money is proposed to be laid out in a Free-hold Estate ; To find what Annual Rent that Sum will Purchase, at any given Rate of Interest *per Cent.* &c.

Rule. $\left\{ \begin{array}{l} \text{Multiply the proposed Sum to be laid out in the} \\ \text{Estate, with } R - 1 ; \text{ viz. with the Ratio of} \\ \text{the given Rate of Interest per Cent. and the} \\ \text{Product will shew the Yearly Rent required.} \end{array} \right.$

Example in Yearly Rents.

Suppose 3560 l. were proposed to be laid out in the Purchase of a Free-hold Estate ; What Annual Rent would it Buy, allowing the Purchaser but Four per Cent. per Annum Interest.

In this Example there is given $P = 3560$ $R = 1,04$

To find U the Yearly Rent.

Thus $P = 3560$ } Multiply.
And $R - 1 = 0,04$ }

Product is $142,4 = 142 \text{ l. } 8 \text{ s.} = U$ the Yearly Rent required.

Or the same may be found by *Logarithms*.

Thus $P = 3560$ its Log. is $3,551450$ } Add
 $R - 1 = 0,04$ its Log. is $8,602060$ }

The Sum is the Logarithm. $2,153510$ of $142,4 = U$
Viz. $142 \text{ l. } 8 \text{ s.}$ will be the Rent required, if it be paid but once a Year ; which is just 25 Years Value.

But if the Rent must be paid Half-yearly, or Quarterly, &c.

Then $\left\{ \begin{array}{l} \text{Instead of Multiplying the Sum proposed to be} \\ \text{Laid out in the Purchase, with } R - 1 \text{ (as above)} \\ \text{you must take such a part of the Log. of } R \\ \text{(viz. of the Amount of } 11) \text{ as the Times of} \\ \text{Payment require, and the Number answering to} \\ \text{that part being made less by 1, must be the} \\ \text{Multiplier instead of } R - 1. \end{array} \right.$

Take

Take the same Example in Half-yearly Rents,

That is, Let there be given $P = 3560$ and $R = 1,04$
To find U the Half-yearly Rent.

First $R = 1,04$ the $\frac{1}{2}$ of its Log. is $0,008516$ whose
Number is $1,0198$. Less 1 , is $0,0198$ the Multiplicat.
Then $3560 \times 0,0198 = 70,488$ viz. 70 l. 9 s. 9 d. $= U$
the Half-yearly Rent.

Or, having found the New Multiplier, as above;

Then $P = 3560$ its Log. is $3,551450$ } Add
Multiplicat. $0,0198$ its Log. $8,296665$ }

The Sum is the Logarithm, $1,848115$, of $70,488 = U$;
viz. 70 l. 9 s. 9 d. is the Half-yearly Rent, As before.

Example. 3. For Quarterly Rents.

Suppose the same Sum of 3560 l. were proposed to be
Laid out in the Purchase of a Free-hold Estate; and it
were required to find what Quarterly Rent it would Buy,
at the Rate of Four per Cent. &c.

Here is also given $P = 3560$ and $R = 1,04$. To find U
the Quarterly Rent. Thus,

First $R = 1,04$ the $\frac{1}{4}$ of its Log. is $0,004258$ whose
Number is $1,009861$ and $1,009861 - 1 = 0,00986$ the
New Multiplier.

Then $3560 \times 0,00986$ gives $35,1$ viz. 35 l. 2 s. $= U$
the Rent to be paid every Quarter; which is Less than
the $\frac{1}{4}$ of the Annual Rent by 10 s. per Quarter.

Whence it plainly appears, that the Less the Inter-
vals of the Times are, betwixt the Payments of the
Rents, the More valuable the Purchase is; & vice
versa.

Case. 3. The Annual Rent of any Free-hold Estate,
and the Sum that is paid for it, Being known; To find
what Rate of Interest per Cent. is allow'd to the
Purchaser.

Rule

Rule. $\left\{ \begin{array}{l} \text{Divide the Annual Rent by the Sum that is paid} \\ \text{for the Purchase, and the Quotient will shew the} \\ \text{Ratio of the Rate of Interest per Cent.} \end{array} \right.$

Example.

Suppose a Free-hold Estate of 250 l. per Annum, Cost 4166 l. 13 s. 4 d. What Rate of Interest per Cent. is allow'd the Purchaser?

In this Example there is given $U=250$ $P=4166,667$
To find R , or rather $R-1$, the Ratio of the Rate.
Thus $4166,667 \div 250,000 (0,06=R-1)$. Then it will
be As 1 : 15 to 0,06 :: So is 100 : To 6, the Rate
of Interest per Cent. As was requir'd.

Or by Logarithms; Thus,

First $U=250$ its Log. is 2.397940 } Subtract
And $P=4166,667$ its Log. 3.619789 }

The Difference is the Log. 8.778151 of $0,06=R-1$
Or to the last Log. Add this 2 000000 of 100.

The Sum will be the Log. 0.778151 of 6 the Rate of
Interest per Cent. As before.

Thus far concerning such Questions that are about
Interest, Annuities, or Leases, &c. as are limited by
any assign'd Time, and Real Estates that are Pur-
chased to Continue for Ever; And 'tis only such that
can be Computed by any Certain Rules: However, it
may not (I presume) be unacceptable to the Reader if I
proceed a little further, and insert a brief Account of
such Helps to Estimate the Values of Annuities, or
Leases for Lives, as are proposed by Two very Inge-
nious Persons.

Section 2. How to Estimate the Value of Annuities, or Leases for Lives.

The Way generally used in Buying of Annuities, or
Letting of Leases for Lives, is only by an imaginary
Valuation,

Valuation, grounded upon Custom, and not upon any Consideration that is had to the Age of the Persons whose Lives are to be inserted in the Lease, &c. 'Tis true indeed, that there can be no certain Rules prescrib'd for their true Valuation, because the Lives of all Mankind are uncertain; and 'tis possible, and daily seen, that a Young Man may Die before one of a greater Age: But yet there is a greater Probability of a Young Man's Living longer than an Old one; And not only so, but there's a Proportional Likelihood of the Length of Mens Lives, according to their different Degrees of Age; The which being duly considered, must needs be found of good Use in Estimating the Values of Annuities or Leases for Lives, much better than by a meer Gueffing only, as usual: And that such a Proportional Likelihood is worth the Consideration, will appear from what follows.

1. Sir William Petty, in a Discourse of his, made before the Royal Society (*Anno* 1674.) concerning the Use of **Duplicate Proportion**, doth amongst other things, apply it to the Life of Man, and its Duration, Thus,

It is found by Experience (*saieth he*) that there are more Persons Living of between 16 and 26 Years old, than of any other Age or Decad of Years in the whole Life of Man (*which David and Experience say to be between 70 and 80 Years*) The Reasons whereof are not abstruse; *viz.* because those of 16 have passed the Danger of *Teeth, Convulsions, Worms, Rickets, Meazles, and Small-Pox* for the most part: And that those of 26 are scarce come to the *Gout, Stone, Dropsie, Palsies, Lethargies, Apoplexies*, and other Infirmities of Old Age. Now whether these be sufficient Reasons, is not the present Enquiry; but taking the aforementioned Assertions to be true, I say, that the Roots of every Number of Men's Ages under 16 (*whose Root is 4*) compared with the said Number 4, doth shew the Proportion of the Likelihood of such Men's reaching to 70 (*or 80*) Years of Age.

As

As for Example; 'Tis Four times more likely, that one of 16 Years old should live to 70, than a new-born Babe. 'Tis Three times more likely, that One of 9 Years old should attain the said Age of 70, than the said Infant. Moreover, 'tis Twice as likely, that One of 16 should reach that Age, as that One of 4 Years old should do it; and One Third more likely, than for One of 9.

On the other Hand, 'Tis Five to Four, that One of 25 Years old will Die before One of 16; and Six to Five, that One of 36 will Die before One of 25; and Three to Two, that the same Person of 36 shall Die before him of 16; And so forward, according to the Roots of any other Year of the declining Age, compared with a Number between 4 and 5 (*viz. with 4,58*) which is the Root of 21, the most hopeful Year for Longevity, as the Mean between 16 and 26; and is the Year of Perfection, according to the Sence of *Our Law*, and the Age for whose Life a *Lease* is most valuable.

To prove all which, I can produce the Accompts of every Man, Woman, and Child within a certain Parish of 330 Souls; All which particular Ages being Added together, and the Sum Divided by the whole Number of Souls, made the *Quotient* between 15 and 16; which I call the Age of that Parish, or Numerous *Index* of Longevity there.

Thus you have a Learned Gentleman's Opinion concerning the Likelihood of the Length of Men's Lives, according to the Rules of *Duplicate Proportion*; which was a very ingenious Thought of His: But I must beg Pardon, that I cannot agree with Him in that Part of it, which asserts, That 21 Years is the Age for whose Life a Lease is the most valuable: For although 'tis true, that according to our Law, a Man is said to be then at his Perfect or Full Age, as to the Enjoyment of an Estate, or Managing his Affairs without a Guardian, &c. Yet I should rather adhere to the Close of His Discourse, wherein He says, That He found, that

that if the Sum of all the Ages of the 330 Souls (*in a certain Parish*) being Divided by their Numbers, made the Quotient between 15 and 16. Whence I take 16 to be the Age, for whose Life a Lease is the most valuable; And upon that Supposition I have Calculated the following Table, according to the *aforesaid Rules of Duplicate Proportion.*

<i>Ages</i>	<i>Year's Purch.</i>	<i>Ages</i>	<i>Year's Purch.</i>	<i>Ages</i>	<i>Year's Purch.</i>
1	2,50	26	7,20	51	5,04
6	5,51	31	6,46	56	4,81
11	7,46	36	6,00	61	4,60
16	9,00	41	5,62	66	4,43
21	7,85	46	5,31	71	4,27

This Table shews, by Inspection, the Value of every Five Years of any single Life, from the Birth to 71 Years old. Supposing that any Annuity, or Lease, &c. is really Worth Nine Year's Purchase for One Life; which is according to the Rate that the Annuities settled by a late Act of Parliament for Lives, were valued at, and from thence the rest are computed.

As for Instance; An Annuity, or Lease of 100 *l.* per Ann. is by this Table Worth 900 *l.* (*viz.* 9 x 100) for the Life of One that is about 16 Years old; and 'tis Worth but 600 *l.* for the Life of One that is 36 Years old, and but 250 *l.* for the Life of a Child of a Year old: And so on for any other proposed Annuity, or Lease, and Age of the Person for whose Life it is to be Purchased: Which is so easie to apprehend, that I need say no more of its Uses, but proceed to the next.

2. The Ingenious and Great Mathematician, Mr. Edmund Halley, *Savilian Professor of Geometry* in the University of Oxford, hath (*in the Philosophical Transactions, Numb. 496.*) given us a most Excellent Essay to Estimate the Degrees of the Mortality of Mankind, which he deduced with a great deal of Art and Labour, from
curious

curious *Tables* of the *Births* and *Funerals* that were in *Breslaw*, the Capital City of the Province of *Silesia* in *Germany*, for Five Years successively, viz. from the 1687, to 1691 inclusive, drawn up Monthly by one *Doctor Newman* of that City, and communicated to the *Royal Society* here; from whence *Mr. Halley* hath form'd the following *Table*; which is, without doubt, very carefully and exactly done, and does give a more perfect Account of the State and Condition of Mankind in respect of their *Chances* of *Mortality* at all *Ages*, and consequently how to Estimate the *Values* of *Annuities*, or *Leases* for *Lives*, better than by any other Method ever proposed before it. The *Table* does shew the *Number* of *Persons* that were Living in their respective *Ages* Current annexed thereto, as follows.

Age Curr.	Persons Living.	Age Curr.	Persons Living.	Age Curr.	Persons Living.	Age Curr.	Persons Living.
1	1000	22	586	43	417	64	202
2	855	23	579	44	407	65	192
3	798	24	573	45	397	66	182
4	760	25	567	46	387	67	172
5	732	26	560	47	377	68	162
6	710	27	553	48	367	69	152
7	692	28	546	49	357	70	142
8	680	29	539	50	346	71	131
9	670	30	531	51	335	72	120
10	661	31	523	52	324	73	109
11	653	32	515	53	313	74	98
12	646	33	507	54	302	75	88
13	640	34	499	55	292	76	78
14	634	35	490	56	282	77	68
15	628	36	481	57	272	78	58
16	622	37	472	58	262	79	49
17	616	38	463	59	252	80	41
18	610	39	454	60	242	81	34
19	604	40	445	61	232	82	28
20	598	41	436	62	222	83	23
21	592	42	427	63	212	84	20

This

This *Table* may be Applied to very many Uses, but I shall only insert what may suffice for the present Purpose.

1. The First Use is to shew the Differing Degrees of *Mortality*, or rather *Vitality* in all *Ages*; For if the Number of Persons of any Age Remaining after One Year, be Divided by the *Difference* between that and the Number of the Age proposed, it shews the Odds that there is, that a Person of that Age does not Die in One Year.

As for Instance, a Person of 25 Years of Age has the Odds of 560 to 7 (*viz.* 80 to 1) that he does not Die in a Year: Because that of 567 Persons Living of 25 Years of Age, there do Die no more than 7 in a Year, Leaving 560 of 26 Years Old: Or,

2. If it be required to find the Odds, that any Person does not Die before he attain to any proposed Age. Then,

Take the Number of the Remaining Persons of the Age proposed, and Divide it by the *Difference* between it and the Number of those of the Age of the Party proposed; and that shews the Odds there is between the Chances of the Parties Living, or Dying.

As for Instances; What is the Odds that a Man of 40 may Live 7 Years? Take the Number of Persons of 47 Years, which in the *Table* is 377, and Subtract it from the Number of Persons of 40 Years, which is 445 and the *Difference* is 68, *viz.* $445 - 377 = 68$ which shews that the Persons Dying in the 7 Years are 68, and that 'tis 377 to 68, or $5\frac{1}{2}$ to 1, that a Man of 40 does Live 7 Years: And the like for any other Number of Years.

3. And if it be required to find at what Number of Years, it is an even Lay that a Person of any proposed Age shall Die, this *Table* readily performs it: For if the Number of Persons Living of the Age proposed be halv'd, it will be found by the *Table* at what Year the said Number is Reduc'd to Half by *Mortality*; and that is the Age, to which it's an even Wager, that a Per-

son of the *Age* proposed shall arrive to before he Die.

As for Instance, a Person of 30 Years of *Age* is proposed; the *Number* of that *Age* is 531, the Half of it is 265; which *Number* I find to be between 57 and 58 Years: So that a Man of 30 may reasonably expect to Live between 27 and 28 Years.

4. By what has been said, the *Price of Insurance* upon *Lives* ought to be Regulated, and the *Difference* is discovered between the *Price of Insuring the Life* of a Man of 20 and One of 50 Years Old.

For Example; It being 100 to 1, that a Man of 20 Dies not in a Year, and but 38 to 1, for a Man of 50 Years of Age.

5. And upon these *Proportions* depend the *Valuation* of *Annuities* for *Lives*: For it's plain, that the Purchaser ought to pay only for such a Part of the *Annuity*, as he hath Chances that he is Living; and this ought to be Computed Yearly, and the Sum of all those Yearly *Values* being Added together, will be the *Value* of the *Annuity* for the Life of the Person proposed. Now the present *Value* of Money payable after any Term of Years, may be easily Computed by Case 2. Page 54. at any given Rate of Interest.

As the *Number* of Persons Living after that Term of Years; to the *Number* Dead; So are the Odds that any one Person is Alive or Dead.

Then { And by consequence, As the Sum of both, or the *Number* of Persons Living of the *Age* first proposed: To the *Number* Remaining after so many Years (both being given by the Table): : So is the present *Value* of the Yearly Sum payable after the Term proposed: To the Sum which ought to be paid for the Chance the Person has to Enjoy such an *Annuity* so many Years. And this being repeated for every Year of the Person's Life, the Sum of all the present *Values* of those Chances is the true *Value* of the *Annuity*.

-Now

Now because the Work of these Proportions, is somewhat troublesome to perform; the *Ingenious Author* hath been so kind as to take the Pains (*which was not a little*) to Calculate the following Table; which shews the Value of *Annuities*, or *Leases*, &c. (at the Rate of 6 per Cent.) for every Fifth Year of Age to the Seventieth.

Ages	Year's Purch.	Ages	Year's Purch.	Ages	Year's Purch.
1	10,28	25	12,27	50	9,21
5	13,40	30	11,72	55	8,51
10	13,44	35	11,12	60	7,60
15	13,33	40	10,57	65	6,54
20	12,78	45	9,91	70	5,32

This Table being of the same Nature with that in *Pag. 107*, there needs no other Explanation or Example to shew its Use, than what has been already said about that Table: Only here I must again beg leave to give my Opinion about the *Difference* of the *Proportions* in the Two Tables; which is, that as the Table in *Page 107*, may not be thought a sufficient Guide to be depended upon in Estimating the Length of Men's Lives, &c. because its only deduced from the bare Rules of Art, *viz.* that of *Duplicate Proportion*; so on the other Hand, I doubt the Estimates of the Value of any Annuity taken from this Table, will be found too great in this Country, (*viz.* in England) which I much fear hath not so Good or *Salubrious* an Air, as that at the City of *Breslaw*, from whence these Calculations are drawn: But if in Imitation hereof the Curious in other Cities and Large Towns would attempt something of the same Nature; Then without all doubt this Method of Estimating the Probability of the Length of Mens's Lives, would prove the Best, and become more universally Useful than can be expected from this one single Instance, more especially if such Observations were continued for any considerable Time, as 20, or 30 Years suc-

cessively. And then it would be well worth the Time and Pains to Calculate proper *Tables*, of the *Value* of *Annuities*, or *Leases*, &c. both for Two, and for Three Lives, according to the following Rules; which are deduced from the former *Table*.

And First, Two Lives being proposed, their Values may be thus found.

If the *Number* of *Chances* of each single Life, found in the *Table*, be Multiplied together, the *Product* is the *Chances* of those Two Lives; And after any certain Term of Years, the *Product* of the Two Remaining Sums, will be the *Chances* that both the Persons are Living: And the *Product* of the *Differences*, being the *Numbers* of the *Dead* of both Ages, will be the *Chances* that both the Persons are *Dead*.

Then { As the *Product* of the Two Numbers in the *Table* for the Two Ages proposed: Is to the *Difference* between that *Product* and the *Product* of the Two Numbers of the Persons Deceased in any given Space of Time :: So is the *Value* of a Sum of Money to be paid after that Time: To the *Value* thereof under the Contingency of Mortality.

And, if Three Lives are proposed, to find the Value of an Annuity during the Continuance of those Lives;

Then { As the *Product* of the continual Multiplication of the Three Numbers in the *Table*, answering to the Ages proposed: Is to the *Difference* of that *Product*, and the *Product* of the Three Numbers of the Deceased of those Ages in any given Term of Years :: So is the present *Value* of a Sum of Money, to be paid certainly after so many Years: To the present *Value* of the same Sum to be paid, provided One of those Three Persons be Living at the Expiration of that Term of Years.

These Proportions being Yearly repeated, the Sum of all those present Values will be the Value of the Annuity granted for those Lives.

These

These Rules are Explain'd at Large by their Author, both *Algebraically* by Letters, and by *Geometrical Figures*: And he also proceeds on (*by the same Method*) to Compute the present *Value* of the *Reversion* of any *Annuity*, or *Lease*, either of One Life after another; or after Two Lives, &c. The which being not only too long a Discourse to be inserted in this small *Treatise*, but also too Difficult a Piece a Work for any Ordinary Arithmetician to undertake, I have therefore omitted it, and refer those that are Curious, and desire further Satisfaction therein, to the aforementioned *Philos. Transf. Numb. 196.* and shall only Add this serious Observation, *Viz.* How unjustly we generally Repine at the Shortness of our Lives, and think ourselves Wrong'd if we attain not to Old Age; Whereas it appears, that the One Half of those that are *Born*, do *Die* in *Seventeen Year's Time*. For by the aforesaid Bills of *Mortality* at *Breslaw* it was found, that 1238 were in that Time Reduced to 616. So that instead of *Murmuring* at what we call a *Short Life*, we ought with Patience and Unconcern to submit to that Dissolution which is the necessary Condition of our perishable Materials, and of our nice and frail Structure and Composition; And to account it as a great *Blessing* that we have survived, (perhaps by many Years) that *Period* of Life, whereat the One Half of the whole Race of Mankind does not Arrive.

Having now gone through all the General Cases of Interest, and Annuities, &c. I design'd to have concluded here; But because the Business of *Rebate* or *Discompt* of Money paid before the Time it becomes Due, comes often into Practice upon several Occasions, and being but just touch'd upon in Page 24 and 54; altho' even what is there done, being duly consider'd, might be sufficient; yet lest I should be thought too short or remiss in so useful a Part of Interest as that is: It may be convenient to proceed a little further, and lay down particular Rules for that Purpose.

C H A P. VI.

To find the **Rebate** or **Discompt** of any proposed Sum, &c. and the true **Equated** Time of several Payments ; either at *Simple*, or *Compound Interest*.

SECT. I. To find the true *Discompt* of any Sum, at any Rate of *Simple Interest*.

Suppose $\left\{ \begin{array}{l} S = \text{the Sum proposed to be Discompted for;} \\ T = \text{the Interval of Time it becomes Due;} \\ R = \text{the Ratio of the Rate of Interest per Cent.} \end{array} \right.$
And $D = \text{the Discompt or Rebate sought.}$

Then it will be $\frac{TRS}{TR + 1} = D.$

Which in Words gives this following Rule.

Rule. $\left\{ \begin{array}{l} \text{First Multiply the given Time with the Ratio} \\ \text{of the Rate of Interest, and to their Product} \\ \text{Add 1, that Sum will be the Divisor.} \\ \text{Next Multiply the first Product and the Sum} \\ \text{that's to be Discompted for, and that Product} \\ \text{will be the Dividend; The Quotient arising from} \\ \text{abence will shew the Discompt requir'd.} \end{array} \right.$

Example.

Let it be required to find what *Discompt* ought to be allowed for 3560 l. if it be paid 273 Days before it becomes Due, at the Rate of Six per Cent. per Annum *Simple Interest*?

Here is given $S = 3560$, $T = 0,74794$ (found by the Table in Pag. 29.) And $R = 0,06$ To find D .

First $0,74794 \times 0,06 = 0,044876$

And $0,044876 + 1$ is $1,044876$ for the Divisor.

Next $0,044876 \times 3560 = 159,75856$ the Dividend.

Then $1,044876 \div 159,75856 (152,897 = D.$

That is 152 l. 18 s. 1/2 p. will be the *Discompt* required.

The

and Equation of Payments. 115

The same perform'd by *Logarithms*.

First, $T = 0,74794$ its Log. 9.873867 } Add.
And $R = 0,06$ its Log. is 8.778151 }

The Sum is the Log. 8.652018 of $0,044876$
Next $S = 3560$ its Log. is 3.551450 Add to the last

Sum 2.203468 } Subtract
 $0,044876 + 1 = 1,044876$ Log. 0.019066 }

There Remains the Logar. 2.184402 of $152,897$
That is 152 l. 18 s. As above, which being Subtracted
from 3560 l. the proposed Sum, there will Remain
 3407 l. 2 s. the Sum to be paid in ready Money; as
may be easily proved by making it a Principal, and
then finding what it would Amount to in 273 Days
at 6 per Cent. which by *Case 1. Page 22.* will be just
 3560 l. Consequently the Discount is truly found.

Secondly, To find the true Rebate or Discount of
any Sum, at any given Rate of Compound Interest.

Let us suppose S , t , and D , to represent the same
Parts of the Question as before; And R , to denote
the Amount of 1 l. &c. as in *Page 53.*

$$\text{Then will } \left\{ \frac{SR^t - S}{R^t} = D. \right.$$

And from this Equation is deduced the following
Rule.

{ Multiply the Logarithm of R the Amount of
1 l. at the given Rate of Interest, with the Time,
and the Number which belongs to their Product
will be the Divisor.

Rule. { From that Number Subtract 1 , then Mul-
tiply the Remainder with the proposed Sum that's
to be Discounted for, and that Product will be the
Dividend; the Quotient arising from thence will
shew the Discount required.

Example.

Suppose it were required to find what Discount or
Rebate must be allow'd for the Payment of 956 l. 10 s.
Nine

Nine Months (viz. $\frac{3}{4}$ of a Year) before it becomes Due, at the Rate of Five and a Half per Cent. Compound Interest.

In this Question there is given $S = 956,5$ $t = 0,75$ and $R = 1,055$ To find D .

First $R = 1,055$ its Log. is 0.023252 } Multiply
And $t = 0,75$

The Product is the Log. 0.017439 of $1,04097$ the Divisor: And $1,04097 - 1 = 0,04097$ will be the Multiplier: And $956,5 \times 0,04097 = 39,1878$ the Dividend. Then $1,04097 \mid 39,1878$ ($37,645 = D$; viz. 37 l. 12 s. $10\frac{3}{4}$ d. is the Discompt required.

Or, having first found the Divisor, and by it the Multiplier, As above; Then the rest of the Work is very easily perform'd by *Logarithms*.

Thus $S = 956,5$ its Logar. is 2.980685 } Add
Multiplier $0,04097$ its Log. 8.612466

Sum 1.593151 } Subtract
Divisor is $1,04097$ its Log. is 0.017439

There Remains the Logarith. 1.575712 of $37,645$ Viz. 37 l. 12 s. $10\frac{3}{4}$ d. is the Discompt required, As above; which being taken from the 956 l. 10 s. leaves 918 l. 17 s. $1\frac{1}{4}$ d. the Sum which is to be paid in ready Money; as may be tried, by making it a Principal, and then finding what it would Amount to in $\frac{3}{4}$ of a Year at the Rate of Five and a Half per Cent. the which, by *Case 1. Page 53.* will be found to be just 956 l. 10 s. Therefore, &c.

If these Two Rules, and their Examples, be a little consider'd, I presume it will be very easie to conceive how to find the true Discompt. or Rebate of any One propos'd Sum of Money, Due at the End of any given Interval of Time, and any given Rate, either of Simple, or Compound Interest, according as the Question is propos'd.

And if it be required to find the whole Discompt of several Sums, Due at the End of several different Intervals

vals of Time, it is but Computing them at so many several Operations, and then the Sum of all those particular Discmpts being taken from the Total Sum of all the Debts, will leave the true Discmpt requir'd.

As for Instance ;

Suppose A were indebted to B 750 l. to be paid at Three several Payments, in this manner ; viz. 250 l. at the End of One Year and a Half, 100 l. to be paid at the End of Two Years, and 400 l. at the End of Four Years ; The Question is, To find how much of the 750 l. B, ought to Rebate or Discmpt, at Six per Cent. per Annum Simple Interest, to have his Whole Debt discharged by A in present Money ?

According to the Data's in this Question, the particular Discmpts (found by the First Rule in this Section) will be these following :

The Discmpt of $\left. \begin{array}{l} 250 \text{ l. for 1,5 Year is } 20,6423 \\ 100 \text{ l. for 2 Years is } 10,7142 \\ 400 \text{ l. for 4 Years is } 27,4193 \end{array} \right\} \text{Add}$

Consequently the whole Discmpt is 108,7758 Then if from 750 l. the whole Debt, there be Subtracted 108,7758 the whole Discmpt, there will Remain 641,2242 = 641 l. 4 s. 5 $\frac{1}{2}$ d. which is the true Sum that A, must give to B, in present Money, to be Discharg'd of his Debt.

Again, Suppose the same things given as above, and let it be requir'd to find the particular Discmpts, &c. at Compound Interest.

Then, by the Second Rule in this Section, the Work will stand thus :

The Discmpts of $\left. \begin{array}{l} 250 \text{ l. for 1,5 Year is } 20,9238 \\ 100 \text{ l. for 2 Year is } 11,0003 \\ 400 \text{ l. for 4 Year is } 83,1633 \end{array} \right\} \text{Add}$

Sum is 115,0874

Then 115,0874 Taken from 750 l. Leaves 634,9126 Viz. 634 l. 18 s. 3 d. is now the Sum which A is to give B, in present Money, to be clear of his Debt.

This

This is so plain to be understood, that I presume it's needless to say any more in Proof of it, than what has been already said in Page 115, and 116.

And from hence Naturally flows the following Method of finding the true Equated Time, wherein several Sums, Due at several Intervals of Time, may be paid at one entire Payment, without any Loss, either to the Creditor or Debtor.

Sett. 2. The Equated Time of Payments truly Determin'd.

The usual Rule laid down in divers Treatises of Arithmetick and Interest, &c. for finding of an Equated Time, for the Payment of several Sums of Money Due at the End of unequal Intervals of Time, is to this Effect.

Rule. $\left\{ \begin{array}{l} \text{Multiply every single Sum of Money with the} \\ \text{Time it becomes Due; and Divide the Sum of} \\ \text{those Products by the Total Debt; and the Quotient} \\ \text{will be the true Time (say they) at which all the} \\ \text{Money ought to be paid.} \end{array} \right.$

I shall pass over all the Arguments that are made use of *pro* and *con*, by Mr. John Kersey, in his Book of Arithmetick, and by Sir Samuel Morland, in his Doctrine of Interest, and other Authors, about the Eroneousness of this Rule; As also the Rules they lay down instead of it; and shall only proceed to shew how the true Equated Time may be found, from what hath been already done and proved.

When any Number of Payments are proposed to be paid off at one entire Sum; Then, in order to determine the Equated Time for the Payment of that Sum, you must first find the particular Discompts of all the proposed Payments, whose Times are to be Equated, according to the given Intervals of those Payments, at any Rate, either of Simple, or Compound Interest, as shall be agreed on by the Parties concern'd; and by those Discompts find the present Sum that would Clear the Debt, if it were immediately paid; (*As above.*)

Then

Then $\left\{ \begin{array}{l} \text{If the Sum of all those particular Discounts,} \\ \text{be Divided by the Product of the present Sum} \\ \text{Multiplied with the Ratio of the same Rate of} \\ \text{Interest by which those Discounts were Computed,} \\ \text{the Quotient will shew the true Equated Time} \\ \text{required at Simple Interest.} \end{array} \right.$

For an Example of this Rule ;

Let it be requir'd to find the true Equated Time, wherein the aforesaid three Sums, Due from A, to B (As in the last Instance) may be safely paid without Loss to either, &c.

There the Sum of all the particular Discounts at Six per Cent. Simple Interest, is found to be 108,7758 and $750 - 108,7758 = 641,2242$ the present Sum, if it were to be immediately paid.

Then, by the Rule above, $641,2242 \times 0,06 = 38,47345$ will be the Divisor ; And 108,7758 the Dividend. Then $38,47345 \div 108,7758 = 2,8273$ is the Quotient ; That is, 2 Years and 302 Days will be the Time when A, may pay unto B his whole Debt of 750 l. without any Loss to either of them.

The same found by Logarithms.

Sum of Discounts 108.7758 its Log. 2.036544

Present Worth is 641,2242 its Log. 2.807010 } Add
Ratio of the Rate is 0,06 its Log. 8.778151 }

From the 1st Log. Substr. this Log. 1.585161 Sum ;

There Remains the Logarithm 0.451383 of 2.8273 Viz. 2 Years and 302 Days will be the true Equated Time required by the Question ; As before.

For if the Discounts be truly found, then it cannot be denied, but that the Present Worth of the whole Debt may by them be truly found ; And, I say, if that Present Worth be made a Principal, and the Equated Time (as here found) be made the Time of that Principal's being forborn at the same Rate of Interest, it will be found (per Case 1. Pag. 23) to Amount to just the whole Debt ; Therefore, the Equated Time is here truly found. And

And if it be required to find the Equated Time at any proposed Rate of Compound Interest, you must proceed in the same manner to find all the particular Discounts of the given Sums, and by them the present Sum that would Clear the Debt, if it were immediately paid; As in Page 115.

Then { If the Log. of the present Sum be taken from the Log. of the Total Sum of all the Debts, and the Remainder be Divided by the Log. of R, the Amount of 1 l. at the same Rate of Interest by which those Discounts were Computed; The Quotient will shew the Equated Time requir'd.

As for Example.

Suppose it were requir'd to find the Equated Time at Compound Interest, wherein the 750 l. Due from A to B, in the same manner as before, may be paid at one entire Payment, without Loss, &c.

Then the Sum of all the particular Discounts, found at 6 per Cent, Compound Interest, is 115,0874
And $750 - 115,0874 = 634,9126$ is the present Sum that would Discharge the Debt, if it were immediately paid, as in Page 117. Then, by the Rule above, it will be 750 its Logarithm is 2.875061 } Subst.
Present Sum 634,9126 its Log. is 2.802714 }

$R = 1,36$ its Log. is 0.025306) 0.072347 (2,8588
That is, 2 Years, and 313 Days, is here found to be the Equated Time that A must Pay the 750 l. unto B, at one entire Payment; which is but 11 Days more than the Time found at Simple Interest.

And the Truth of the Equated Time thus found at Compound Interest, may be easily proved by the Help of Case 1. Page 53, &c. in the same manner as that of Simple Interest was in the last Page.

F I N I S.

A Table of Logarithms, for Numbers
Increasing in their proper Order,
from 1, To 10000.
With their Differences.

<i>Num</i>	<i>Logarithb.</i>	<i>Num</i>	<i>Logarithb.</i>	<i>Num</i>	<i>Logarithb.</i>
1	0.000000	34	1.531479	67	1.826075
2	0.301030	35	1.544068	68	1.832509
3	0.477121	36	1.556302	69	1.838849
4	0.602060	37	1.568202	70	1.845098
5	0.698970	38	1.579784	71	1.851258
6	0.778151	39	1.591065	72	1.857332
7	0.845098	40	1.602060	73	1.863323
8	0.903090	41	1.612784	74	1.869232
9	0.954242	42	1.623249	75	1.875061
10	1.000000	43	1.633468	76	1.880814
11	1.041393	44	1.643453	77	1.886491
12	1.079181	45	1.653212	78	1.892095
13	1.113943	46	1.662758	79	1.897627
14	1.146128	47	1.672098	80	1.903090
15	1.176091	48	1.681241	81	1.908485
16	1.204120	49	1.690196	82	1.913814
17	1.230449	50	1.698970	83	1.919078
18	1.255272	51	1.707570	84	1.924279
19	1.278754	52	1.716003	85	1.929419
20	1.301030	53	1.724276	86	1.934498
21	1.422219	54	1.732394	87	1.939519
22	1.342423	55	1.740363	88	1.944483
23	1.361728	56	1.748188	89	1.949390
24	1.380211	57	1.755875	90	1.954242
25	1.397940	58	1.763428	91	1.959041
26	1.414973	59	1.770852	92	1.963788
27	1.431364	60	1.778151	93	1.968483
28	1.447158	61	1.785330	94	1.973128
29	1.462398	62	1.792392	95	1.977724
30	1.477121	63	1.799341	96	1.982271
31	1.491362	64	1.806180	97	1.986772
32	1.505150	65	1.812913	98	1.991226
33	1.518514	66	1.819544	99	1.995635

A Table of Logarithms,

Num	0	1	2	3	4	Diff
100	000000	000434	000868	001301	001734	432
101	004321	004751	005181	005609	006038	428
102	008600	009026	009451	009876	010300	424
103	012837	013259	013679	014100	014520	420
104	017033	017451	017868	018284	018700	416
105	021189	021603	022016	022428	022841	411
106	025306	025715	026124	026533	026942	408
107	029384	029789	030195	030600	031004	404
108	033424	033826	034227	034628	035029	401
109	037426	037825	038223	038620	039017	397
110	041393	041787	042182	042575	042969	393
111	045323	045714	046105	046495	046885	390
112	049218	049606	049993	050380	050766	386
113	053078	053463	053846	054230	054613	383
114	056905	057284	057666	058046	058426	379
115	060698	061075	061452	061829	062206	376
116	064458	064832	065206	065580	065953	373
117	068186	068557	068928	069298	069668	370
118	071882	072250	072617	072985	073352	366
119	075547	075912	076276	076640	077004	364
120	079181	079543	079904	080266	080626	361
121	082785	083144	083503	083861	084219	357
122	086360	086716	087071	087426	087781	355
123	089905	090258	090611	090963	091315	352
124	093422	093772	094122	094471	094820	349
125	096910	097257	097604	097951	098297	347
126	100371	100715	101059	101403	101747	344
127	103804	104145	104487	104828	105169	341
128	107210	107549	107888	108227	108565	338
129	110590	110926	111262	111598	111934	335
130	113943	114277	114611	114944	115278	332
131	117271	117603	117934	118265	118595	330
132	120574	120903	121231	121560	121888	328
133	123852	124178	124504	124830	125156	325
134	127105	127429	127752	128076	128399	323
135	130334	130655	130977	131298	131618	321

from 1, To 10000.

Num	5	6	7	8	9	Di
100	002166	002598	003029	003460	003891	43
101	006466	006894	007321	007748	008174	42
102	010724	011147	011570	011993	012415	42
103	014940	015360	015779	016197	016615	42
104	019116	019532	019947	020361	020775	41
105	023252	023664	024075	024486	024896	41
106	027350	027757	028164	028571	028978	40
107	031408	031812	032216	032619	033021	40
108	035430	035830	036229	036629	037028	40
109	039414	039810	040207	040602	040998	39
110	043362	043755	044148	044540	044931	39
111	047275	047664	048053	048442	048830	39
112	051152	051538	051924	052309	052694	38
113	054996	055378	055760	056142	056524	38
114	058805	059185	059563	059942	060320	37
115	062582	062958	063333	063708	064083	37
116	066326	066698	067071	067443	067814	37
117	070038	070407	070776	071145	071514	37
118	073718	074085	074451	074816	075182	36
119	077368	077731	078094	078457	078819	36
120	080987	081347	081707	082067	082426	36
121	084576	084934	085291	085647	086004	35
122	088136	088490	088845	089198	089552	35
123	091667	092018	092370	092721	093071	35
124	095169	095518	095866	096215	096562	34
125	098644	098990	099335	099681	100026	34
126	102091	102434	102777	103119	103462	34
127	105510	105851	106191	106531	106870	34
128	108903	109241	109578	109916	110253	33
129	112270	112605	112940	113275	113609	33
130	115610	115943	116276	116608	116940	33
131	118926	119256	119586	119915	120245	33
132	122216	122543	122871	123198	123525	32
133	125481	125806	126131	126456	126781	32
134	128722	129045	129368	129690	130012	32
135	131939	132260	132580	132900	133219	32

A Table of Logarithms,

<i>m</i>	0	1	2	3	4	<i>Diff</i>
6	133539	133858	134177	134496	134814	319
7	136721	137037	137354	137671	137987	316
8	139879	140194	140508	140822	141136	314
9	143015	143327	143639	143951	144263	311
10	146128	146438	146748	147058	147367	309
11	149219	149527	149835	150142	150449	307
12	152288	152594	152900	153205	153510	305
13	155336	155640	155943	156246	156549	303
14	158362	158664	158965	159266	159567	301
15	161368	161667	161967	162266	162564	298
16	164353	164650	164947	165244	165541	297
17	167317	167613	167908	168203	168497	295
18	170262	170555	170848	171141	171434	292
19	173186	173478	173769	174060	174351	290
20	176091	176381	176670	176959	177248	289
21	178977	179264	179552	179839	180126	287
22	181844	182129	182415	182700	182985	285
23	184691	184975	185259	185542	185825	282
24	187521	187803	188084	188366	188647	281
25	190332	190612	190892	191171	191451	279
26	193125	193403	193681	193959	194237	278
27	195900	196176	196453	196729	197005	276
28	198657	198932	199206	199481	199755	274
29	201397	201670	201943	202216	202488	272
30	204120	204391	204663	204934	205204	271
31	206826	207096	207365	207634	207904	269
32	209515	209783	210051	210319	210586	267
33	212188	212454	212720	212986	213253	266
34	214844	215109	215373	215638	215902	264
35	217484	217747	218010	218273	218536	262
36	220108	220370	220631	220892	221153	261
37	222716	222976	223236	223496	223755	260
38	225309	225568	225826	226084	226342	258
39	227887	228144	228400	228657	228913	257
40	230449	230704	230960	231215	231470	254
41	232096	232350	232604	232857	233111	252

from 1, To 10000.

Num.	5	6	7	8	9	Diff
136	135133	135451	135768	136086	136403	319
137	138303	138618	138934	139249	139564	316
138	141450	141763	142076	142389	142702	314
139	144574	144885	145196	145507	145818	311
140	147676	147985	148294	148603	148911	309
141	150756	151063	151370	151676	151982	307
142	153815	154119	154424	154728	155032	305
143	156852	157154	157457	157759	158061	303
144	159868	160168	160469	160769	161068	301
145	162863	163161	163459	163758	164055	298
146	165838	166134	166430	166726	167022	297
147	168792	169086	169380	169674	169968	295
148	171726	172019	172311	172603	172895	292
149	174641	174932	175222	175512	175802	290
150	177536	177825	178113	178401	178689	289
151	180413	180699	180986	181272	181558	287
152	183270	183555	183839	184123	184407	285
153	186108	186391	186674	186956	187239	283
154	188928	189210	189490	189771	190051	281
155	191730	192010	192289	192567	192846	279
156	194514	194792	195069	195346	195623	278
157	197281	197556	197832	198107	198382	276
158	200029	200303	200577	200850	201124	274
159	202761	203033	203305	203577	203848	272
160	205475	205746	206016	206287	206556	271
161	208173	208441	208710	208978	209247	269
162	210853	211120	211388	211654	211921	267
163	213518	213783	214049	214314	214579	266
164	216166	216430	216694	216957	217221	264
165	218798	219060	219323	219585	219846	262
166	221414	221675	221936	222196	222456	261
167	224015	224274	224533	224792	225051	260
168	226600	226858	227115	227372	227630	258
169	229170	229426	229682	229938	230193	257
170	231724	231979	232233	232488	232742	254
171	234264	234517	234770	235023	235276	253

A Table of Logarithms,

Nun.	0	1	2	3	4	Diff
172	235528	235781	236033	236285	236537	252
173	238246	238297	238548	238799	239049	250
174	240549	240799	241048	241297	241546	249
175	243038	243286	243534	243782	244030	247
176	245513	245759	246006	246252	246499	246
177	247973	248219	248464	248709	248954	244
178	250420	250664	250908	251151	251395	243
179	252853	253095	253338	253580	253822	242
180	255272	255514	255755	255996	256236	241
181	257679	257918	258158	258398	258637	239
182	260071	260310	260548	260787	261025	238
183	262451	262688	262925	263162	263399	237
184	264818	265054	265290	265525	265761	235
185	267172	267406	267641	267875	268110	234
186	269513	269746	269980	270213	270446	233
187	271842	272074	272306	272538	272770	231
188	274158	274389	274620	274850	275081	230
189	276462	276691	276921	277151	277380	229
190	278754	278982	279210	279439	279667	228
191	281033	281261	281488	281715	281942	227
192	283301	283527	283753	283979	284205	226
193	285557	285782	286007	286232	286456	225
194	287802	288025	288249	288473	288696	224
195	290035	290257	290480	290702	290925	222
196	292256	292478	292699	292920	293141	221
197	294466	294687	294907	295127	295347	220
198	296665	296884	297104	297323	297542	219
199	298853	299071	299289	299507	299725	218
200	301030	301247	301464	301681	301898	217
201	303195	303412	303628	303844	304059	216
202	305351	305566	305781	305996	306210	215
203	307496	307710	307924	308137	308351	213
204	309630	309843	310056	310268	310481	212
205	311754	311966	312177	312389	312600	211
206	313857	314078	314289	314499	314710	210
207	315970	316180	316390	316599	316809	209

From 1, To 10000.

Next	15	16	17	18	19	Diff
172	236789	237041	237292	237544	237795	252
173	239299	239550	239800	240050	240300	250
174	241795	242044	242293	242541	242790	249
175	244277	244524	244772	245019	245266	247
176	246745	246991	247236	247482	247728	246
177	249198	249443	249687	249932	250176	244
178	251688	251881	252125	252367	252610	243
179	254264	254306	254548	254790	255031	242
180	256477	256718	256958	257198	257439	241
181	258877	259116	259355	259594	259833	239
182	261263	261501	261738	261976	262214	238
183	263636	263873	264109	264345	264582	237
184	265996	266232	266467	266702	266937	235
185	268344	268578	268812	269046	269279	234
186	270679	270912	271144	271377	271609	233
187	273001	273233	273464	273696	273927	231
188	275311	275542	275772	276002	276232	230
189	277609	277838	278067	278295	278525	229
190	279895	280123	280351	280578	280806	228
193	282169	282395	282622	282849	283075	227
192	284431	284656	284882	285107	285332	226
193	286681	286905	287130	287354	287578	225
194	288920	289143	289366	289589	289812	224
195	291147	291369	291591	291813	292034	222
196	293363	293583	293804	294025	294245	221
197	295567	295787	296007	296225	296446	220
198	297760	297979	298198	298416	298635	219
199	299942	300160	300378	300595	300813	218
200	302114	302331	302547	302764	302980	217
201	304275	304490	304706	304921	305136	216
202	306425	306639	306854	307068	307282	215
203	308564	308778	308991	309204	309417	213
204	310693	310906	311118	311330	311542	212
205	312812	313023	313234	313445	313656	211
206	314920	315130	315340	315551	315760	210
207	317018	317227	317436	317646	317854	209

A Table of Logarithms,

Num	0	1	2	3	4	Diff
208	318063	318272	318481	318689	318898	209
209	320146	320354	320562	320769	320977	207
210	322219	322426	322633	322839	323046	106
211	324282	324488	324694	324899	325105	205
212	326336	326541	326745	326950	327155	204
213	328380	328583	328787	328991	329194	203
214	330434	330637	330841	331042	331245	202
215	332438	332640	332842	333044	333246	202
216	334454	334655	334856	335057	335257	201
217	336460	336660	336860	337060	337260	200
218	338456	338656	338855	339054	339253	199
219	340444	340642	340841	341039	341237	198
220	342423	342620	342817	343014	343212	197
221	344392	344589	344785	344981	345178	196
222	346353	346549	346744	346939	347134	195
223	348305	348500	348694	348889	349083	194
224	350248	350442	350636	350829	351023	193
225	352183	352375	352568	352761	352954	193
226	354108	354301	354493	354684	354876	192
227	356026	356217	356408	356599	356790	191
228	357934	358125	358316	358506	358696	190
229	359834	360025	360215	360404	360593	189
230	361728	361917	362106	362294	362482	188
231	363612	363800	363988	364176	364363	188
232	365488	365674	365861	366049	366236	187
233	367356	367542	367729	367915	368101	186
234	369216	369401	369587	369772	369958	185
235	371068	371253	371437	371622	371806	184
236	372912	373096	373280	373464	373647	184
237	374748	374932	375115	375298	375481	183
238	376577	376759	376942	377124	377306	182
239	378398	378580	378761	378943	379124	181
240	380211	380392	380573	380754	380934	181
241	382017	382197	382377	382557	382737	180
242	383815	383994	384174	384353	384533	179
243	385606	385785	385964	386142	386321	178

from 1, To 10000.

Num	5	6	7	8	9	Diff
208	319106	319314	319524	319730	319938	209
209	321184	321391	321598	321805	322012	207
210	323252	323458	323665	323871	324077	206
211	325310	325516	325721	325926	326131	205
212	327359	327563	327767	327972	328176	204
213	329398	329601	329805	330008	330211	203
214	331427	331630	331832	332034	332236	202
215	333447	333649	333850	334051	334253	202
216	335458	335658	335859	336059	336260	201
217	337459	337659	337858	338058	348257	200
218	339451	339650	339849	340047	340246	199
219	341435	341632	341830	342028	342225	198
220	343409	343606	343802	343999	344196	197
221	345374	345570	345766	345962	346157	196
222	347330	347525	347720	347915	348110	195
223	349278	349574	349666	349860	350054	194
224	351216	351410	351603	351796	351989	193
225	353147	353339	353532	353724	353916	193
226	355068	355260	355452	355643	355834	192
227	356981	357172	357363	357554	357744	191
228	358886	359076	359266	359456	359646	190
229	360783	360972	361161	361351	361539	189
230	362571	362859	363048	363236	363424	188
231	364551	364739	364926	365113	365301	188
232	366423	366610	366796	366983	367169	187
233	368287	368473	368659	368845	369030	186
234	370143	370328	370513	370698	370883	185
235	371991	372175	372360	372544	372728	184
236	373831	374015	374198	374382	374565	184
237	375664	375846	376029	376212	376394	183
238	377488	377670	377852	378034	378215	182
239	379306	379487	379668	379849	380030	181
240	381115	381296	381476	381657	381837	181
241	382917	383097	383277	383456	383636	180
242	384712	384891	385070	385249	385427	179
243	386499	386677	386856	387034	387212	178

A Table of Logarithms,

Num	0	1	2	3	4	Diff
244	387390	387568	387746	387923	388101	178
245	389166	389343	389521	389697	389875	177
246	390935	391112	391288	391464	391641	176
247	392697	392873	393048	393224	393400	176
248	394452	394627	394802	394977	395152	175
249	396199	396374	396548	396722	396896	174
250	397940	398114	398287	398461	398634	173
251	399674	399847	400020	400192	400365	173
252	401400	401573	401745	401917	402089	172
253	403120	403292	403464	403635	403807	171
254	404834	405005	405176	405345	405517	171
255	406540	406710	406881	407051	407221	170
256	408240	408410	408579	408749	408918	169
257	409933	410102	410271	410440	410608	169
258	411620	411788	411956	412124	412292	168
259	413300	413467	413635	413802	413970	167
260	414973	415140	415307	415474	415641	167
261	416640	416807	416973	417139	417306	166
262	418301	418467	418633	418798	418964	165
263	419956	420121	420286	420451	420616	165
264	421604	421768	421933	422097	422261	164
265	423246	423410	423573	423737	423901	164
266	424882	425045	425208	425371	425534	163
267	426511	426674	426836	426999	427161	162
268	428135	428297	428459	428621	428782	162
269	429752	429914	430075	430236	430398	161
270	431364	431525	431685	431846	432007	160
271	432969	433129	433290	433450	433610	160
272	434569	434728	434888	435048	435207	159
273	436163	436322	436481	436640	436798	159
274	437751	437909	438067	438226	438384	158
275	439333	439491	439648	439806	439964	158
276	440909	441066	441224	441381	441538	157
277	442480	442637	442793	442950	443106	157
278	444045	444201	444357	444513	444669	156
279	445604	445760	445915	446071	446226	155

from 1, To 10000.

Num	5	6	7	8	9	D
244	388279	388456	388634	388811	388989	174
245	390051	390228	390405	390582	390758	175
246	391817	391993	392169	392345	392521	176
247	393575	393751	393926	394101	394176	177
248	395326	395501	395676	395850	396025	178
249	397070	397245	397418	397592	397766	179
250	398808	398981	399154	399327	399501	180
251	400538	400711	400883	401056	401228	181
252	402261	402433	402605	402777	402949	182
253	403978	404149	404320	404492	404663	183
254	405688	405858	406029	406199	406370	184
255	407391	407561	407731	407900	408070	185
256	409087	409257	409426	409595	409764	186
257	410777	410946	411114	411283	411451	187
258	412460	412628	412796	412964	413132	188
259	414137	414305	414472	414639	414806	189
260	415808	415974	416141	416308	416474	190
261	417472	417638	417804	417970	418135	191
262	419129	419295	419460	419625	419791	192
263	420781	420945	421110	421275	421439	193
264	422426	422590	422754	422918	423082	194
265	424064	424228	424392	424555	424718	195
266	425597	425860	426023	426186	426349	196
267	427224	427486	427648	427811	427973	197
268	428944	429106	429268	429429	429591	198
269	430559	430720	430881	431042	431203	199
270	432167	432328	432488	432649	432809	200
271	433770	433930	434090	434249	434409	201
272	435366	435526	435685	435844	436003	202
273	436957	437116	437275	437433	437592	203
274	438542	438700	438859	439017	439175	204
275	440122	440279	440437	440594	440752	205
276	441695	441852	442009	442166	442323	206
277	443253	443419	443576	443732	443889	207
278	444825	444981	445137	445293	445449	208
279	446382	446537	446692	446848	447003	209

A Table of Logarithms;

Num	0	1	2	3	4	Diff
280	447158	447313	447468	447623	447778	155
281	448706	448861	449015	449170	449324	154
282	450249	450403	450557	450711	450865	154
283	451785	451940	452093	452247	452400	153
284	453318	453471	453624	453777	453930	153
285	454845	454997	455149	455302	455454	152
286	456366	456518	456670	456821	456973	152
287	457882	458033	458184	458336	458487	151
288	459392	459543	459694	459845	459995	151
289	460898	461048	461198	461348	461499	150
290	462398	462548	462697	462847	462997	150
291	463893	464042	464191	464340	464489	149
292	465383	465532	465680	465829	465977	149
293	466868	467016	467164	467312	467460	148
294	468347	468495	468643	468790	468938	147
295	469822	469969	470116	470263	470410	147
296	471292	471438	471585	471732	471878	146
297	472755	472903	473049	473195	473341	146
298	474216	474362	474508	474653	474799	146
299	475671	475816	475962	476107	476252	145
300	477121	477266	477411	477555	477700	145
301	478566	478711	478855	478999	479143	144
302	480007	480151	480294	480438	480582	144
303	481443	481586	481729	481872	482016	143
304	482874	483016	483159	483302	483445	143
305	484300	484442	484585	484727	484869	142
306	485721	485863	486005	486147	486289	142
307	487138	487280	487421	487563	487704	141
308	488551	488692	488833	488874	489114	141
309	489958	490099	490239	490380	490520	140
310	491362	491502	491642	491782	491922	140
311	492760	492900	493040	493179	493319	139
312	494155	494294	494433	494572	494711	139
313	495544	495683	495822	495960	496099	139
314	496930	497068	497206	497344	497482	138
315	498311	498448	498586	498724	498862	138

from 1, To 10000.

<i>Num</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>	<i>9</i>	<i>Diff</i>
280	447933	448088	448242	448397	448552	155
281	449478	449633	449787	449941	450095	154
282	451018	451172	451326	451479	451633	154
283	452553	452706	452859	453012	453165	153
284	454082	454235	454387	454540	454692	153
285	455606	455758	455910	456062	456214	152
286	457125	457276	457428	457579	457731	152
287	458638	458789	458940	459091	459242	151
288	460146	460296	460447	460597	460748	151
289	461649	461799	461948	462098	462248	150
290	463146	463296	463445	463594	463744	150
291	464639	464787	464936	465085	465234	149
292	466126	466274	466423	466571	466719	149
293	467608	467756	467904	468052	468200	148
294	469085	469233	469380	469527	469675	147
295	470557	470704	470851	470998	471145	147
296	472025	472171	472318	472464	472610	146
297	473487	473633	473779	473925	474071	146
298	474944	475090	475235	475381	475526	146
299	476397	476542	476687	476832	476976	145
300	477844	477989	478133	478278	478422	145
301	479287	479431	479575	479719	479863	144
302	480725	480869	481012	481156	481299	144
303	482159	482302	482445	482588	482731	143
304	483587	483730	483872	484015	484157	143
305	485011	485153	485295	485437	485579	142
306	486430	486572	486714	486855	486997	142
307	487845	487986	488127	488269	488410	141
308	489255	489396	489537	489677	489818	141
309	490661	490801	490941	491081	491222	140
310	492062	492201	492341	492481	492621	140
311	493458	493597	493737	493876	494015	139
312	494850	494989	495128	495267	495406	139
313	496237	496375	496514	496653	496791	139
314	497621	497759	497897	498035	498173	138
315	498999	499137	499275	499412	499550	138

A Table of Logarithms,

Num	0	1	2	3	4	Diff
316	499687	499824	499962	500099	500236	137
317	501059	501196	501333	501470	501607	137
318	502427	502564	502700	502837	502973	136
319	503791	503927	504063	504199	504335	136
320	505150	505286	505421	505557	505692	136
321	506505	506640	506775	506911	507046	135
322	507856	507991	508125	508260	508395	135
323	509202	509337	509471	508606	509740	134
324	510545	510679	510813	510947	511081	134
325	511883	512017	512150	512284	512417	134
326	513218	513351	513484	513617	513750	133
327	514548	514681	514813	514946	515079	132
328	515874	516006	516139	516271	516403	132
329	517196	517328	517460	517592	517724	131
330	518514	518645	518777	518909	519040	131
331	519828	519959	520090	520221	520352	131
332	521138	521269	521400	521530	521661	131
333	522444	522575	522705	522835	522966	130
334	523746	523876	524006	524136	524256	130
335	525045	525174	525304	525433	525563	129
336	526339	526468	526598	526727	526856	129
337	527630	527759	527888	528016	528145	129
338	528917	529045	529174	529302	529430	128
339	530200	530328	530456	530584	530712	128
340	531479	531607	531734	531862	531989	128
341	532754	532882	533009	533136	533263	128
342	534026	534153	534280	534407	534534	127
343	535294	535421	535547	535674	535800	127
344	536558	536685	536811	536937	537063	127
345	537819	537945	538071	538197	538322	126
346	539076	539202	539327	539452	539578	125
347	540329	540455	540580	540705	540830	125
348	541579	541704	541829	541953	542078	125
349	542825	542950	543074	543199	543323	124
350	544068	544192	544316	544440	544564	124
351	545307	545431	545555	545678	545802	124

from 1, To 10000.

Num	5	6	7	8	9
316	500374	500511	500648	500785	500922
317	501744	501880	502017	502154	502290
318	503109	503246	503382	503518	503654
319	504471	504607	504743	504878	505014
320	505828	505963	506099	506234	506370
321	507181	507316	507451	507586	507721
322	508530	508664	508799	508933	509068
323	509874	510008	510143	510277	510411
324	511215	511348	511482	511616	511750
325	512551	512684	512818	512951	513084
326	513883	514016	514149	514282	514415
327	515211	515344	515476	515609	515741
328	516535	516668	516800	516932	517064
329	517855	517987	518119	518251	518382
330	519171	519303	519434	519566	519697
331	520483	520614	520745	520876	521007
332	521792	521922	522053	522183	522314
333	523096	523226	523356	523486	523616
334	524396	524526	524656	524785	524915
335	525692	525822	525951	526081	526210
336	526985	527114	527243	527372	527501
337	528274	528402	528531	528660	528788
338	529559	529687	529815	529943	530072
339	530840	530968	531095	531223	531351
340	532117	532245	532372	532500	532627
341	533391	533518	533645	533772	533899
342	534661	534787	534914	535041	535167
343	535927	536053	536179	536306	536432
344	537189	537315	537441	537567	537693
345	538448	538574	538699	538825	538951
346	539703	539829	539954	540079	540204
347	540955	541080	541205	541330	541454
348	542203	542327	542452	542576	542701
349	543447	543571	543696	543820	543944
350	544688	544812	544936	545060	545183
351	545925	546049	546172	546296	546419

A Table of Logarithms,

Num	0	1	2	3	4	Diff
352	546543	545666	546789	546913	547036	123
353	547775	547898	548021	548144	548257	123
354	549003	549126	549249	549371	549494	123
355	550228	550351	550473	550595	550717	122
356	551450	551572	551694	551816	551938	122
357	552668	552790	552911	553033	553155	121
358	553883	554004	554126	554247	554368	121
359	555094	555215	555336	555457	555578	121
360	556303	556423	556544	556664	556785	120
361	557507	557628	557748	557868	557988	120
362	558709	558829	558948	559068	559188	120
363	559907	560026	560146	560265	560385	119
364	561101	561221	561340	561459	561578	119
365	562293	562412	562531	562650	562759	119
366	563481	563600	563718	563837	563955	119
367	564566	564784	564903	565021	565139	118
368	565848	565966	566084	566202	566320	118
369	567026	567144	567262	567379	567497	188
370	568202	568319	568436	568554	568671	117
371	569374	569491	569608	569725	569842	117
372	570543	570660	570776	570893	571010	117
373	571709	571825	571942	572058	572174	116
374	572872	572988	573104	573220	573336	116
375	574031	574147	574263	574379	574494	116
376	575188	575303	575419	575534	575650	115
377	576341	576457	576572	576687	576802	115
378	577492	577607	577722	577836	577951	115
379	578639	578754	578868	578983	579097	114
380	579784	579898	580012	580126	580240	114
381	580925	581039	581153	581267	581381	114
382	582063	582177	582291	582404	582518	113
383	583199	583312	583425	583539	583652	113
384	584331	584444	584557	584670	584783	113
385	585451	585573	585686	585799	585912	112
386	586587	586700	586812	586925	587037	112
387	587711	587823	587935	588047	588160	112

from 1, To 10000.

Num	5	6	7	8	9	Diff
352	547159	547282	547405	547529	547652	123
353	548389	548512	548635	548758	548881	123
354	549616	549739	549861	549984	550106	123
355	550840	550962	551084	551206	551328	122
356	552059	552181	552303	552425	552547	122
357	553276	553398	553519	553640	553762	121
358	554489	554610	554731	554852	554973	121
359	555699	555820	555940	556061	556182	121
360	556905	557026	557146	557267	557387	120
361	558108	558228	558349	558469	558589	120
362	559308	559428	559548	559667	559787	120
363	560504	560624	560743	560863	560982	119
364	561698	561817	561936	562055	562174	119
365	562887	563006	563125	563244	563362	119
366	564074	564192	564311	564429	564548	119
367	565257	565376	565494	565612	565730	118
368	566437	566555	566673	566791	566909	118
369	567614	567732	567849	567967	568084	118
370	568788	568905	569023	569140	569257	117
371	569959	570076	570193	570309	570426	117
372	571126	571243	571359	571476	571592	117
373	572291	572407	572523	572639	572755	116
374	573452	573568	573684	573800	573915	116
375	574610	574726	574841	574957	575072	116
376	575765	575880	575996	576111	576226	115
377	576917	577032	577147	577262	577377	115
378	578066	578181	578295	578410	578525	115
379	579212	579326	579441	579555	579669	114
380	580355	580469	580583	580697	580811	114
381	581494	581608	581722	581836	581950	114
382	582631	582745	582859	582972	583085	113
383	583765	583879	583992	584105	584218	113
384	584896	585009	585122	585235	585348	113
385	586024	586137	586250	586362	586475	112
386	587149	587262	587374	587486	587599	112
387	588272	588384	588496	588608	588720	112

A Table of Logarithms,

Num	0	1	2	3	4	Diff
388	588332	588914	589055	589167	589279	112
389	589950	590061	590173	590284	590396	111
390	591065	591176	591287	591398	591510	111
391	592177	592288	592399	592510	592621	111
392	593286	593397	593508	593618	593729	111
393	594393	594503	594613	594724	594834	111
394	595496	595606	595717	595827	595937	110
395	596597	596707	596817	596927	597037	110
396	597695	597805	597914	598024	598134	110
397	598790	598900	599009	599119	599228	109
398	599883	599992	600101	600216	600319	109
399	600973	601082	601190	601299	601408	109
400	602060	602168	602277	602386	602494	109
401	603144	603253	603361	603469	603577	108
402	604226	604334	604442	604550	604658	108
403	605305	605413	605521	605628	605736	107
404	606381	606489	606596	606704	606811	107
405	607455	607562	607669	607777	607884	107
406	608526	608633	608740	608847	608954	107
407	609594	609701	609808	609914	610021	107
408	610660	610767	610873	610979	611086	106
409	611723	611829	611936	612042	612148	106
410	612784	612890	612996	613101	613207	106
411	613842	613947	614053	614159	614264	106
412	614897	615003	615108	615213	615319	105
413	615950	616055	616160	616265	616370	105
414	617000	617105	617210	617315	617420	105
415	618048	618153	618257	618362	618466	105
416	619093	619198	619302	619406	619511	104
417	620136	620240	620344	620448	620552	104
418	621176	621280	621384	621488	621592	104
419	622214	622318	622421	622525	622628	104
420	623249	623353	623456	623559	623663	103
421	624282	624385	624488	624591	624695	103
422	625312	625415	625518	625621	625724	103
423	626347	626449	626546	626648	626751	103

from 1, To 10000.

Num	5	6	7	8	9	Diff
388	589391	589503	589615	589726	589838	112
389	590507	590619	590730	590842	590953	111
390	591621	591732	591843	591955	592066	111
391	592732	592843	592954	593064	593175	111
392	593840	593950	594061	594171	594282	111
393	594945	595055	595165	595276	595386	111
394	596047	596157	596267	596377	596487	110
395	597146	597256	597366	597476	597586	110
396	598243	598353	598462	598572	598681	110
397	599337	599446	599556	599665	599774	109
398	600428	600537	600646	600755	600864	109
399	601517	601625	601734	601843	601951	109
400	602603	602711	602819	602928	603036	109
401	603685	603794	603902	604010	604118	108
402	604766	604874	604982	605089	605197	108
403	605843	605951	606059	606166	606274	107
404	606918	607026	607133	607240	607348	107
405	607991	608098	608205	608312	608419	107
406	609060	609167	609274	609381	609488	107
407	610128	610234	610341	610447	610554	107
408	611192	611298	611405	611511	611617	106
409	612254	612360	612466	612572	612678	106
410	613313	613419	613525	613630	613736	106
411	614370	614475	614581	614686	614792	106
412	615424	615529	615634	615740	615845	105
413	616475	616580	616685	616790	616895	105
414	617524	617629	617734	617839	617943	105
415	618571	618676	618780	618884	618989	105
416	619615	619719	619823	619928	620032	104
417	620656	620760	620864	620968	621072	104
418	621695	621799	621903	622007	622110	104
419	622732	622835	622939	623042	623146	104
420	623766	623869	623973	624076	624179	103
421	624798	624901	625004	625107	625209	103
422	625827	625929	626032	626135	626238	103
423	626853	626956	627058	627161	627263	103

A Table of Logarithms,

num	0	1	2	3	4	Diff
24	627366	627468	627571	627673	627775	102
25	628389	628491	628593	628695	628797	102
26	629410	629511	629613	629715	629817	102
27	630438	630530	630631	630733	630835	102
28	631444	631545	631647	631748	631849	101
29	632457	632559	632660	632761	632862	101
30	633468	633569	633670	633771	633872	100
31	634477	634578	634679	634779	634880	100
32	635484	635584	635685	635785	635886	100
33	636488	636588	636688	636789	636889	100
34	637490	637590	637690	637790	637890	99
35	638489	638589	638689	638789	638888	99
36	639486	639586	639686	639785	639885	99
37	640481	640581	640680	640779	640879	99
38	641474	641573	641672	641771	641871	99
39	642465	642563	642662	642761	642860	99
40	643453	643551	643650	643749	643847	98
41	644439	644537	644636	644734	644832	98
42	645422	645521	645619	645717	645815	98
43	646404	646502	646600	646698	646796	98
44	647383	647481	647579	647676	647774	98
45	648360	648458	648555	648653	648750	97
46	649335	649432	649530	649627	649724	97
47	650308	650405	650502	650599	650696	97
48	651278	651375	651472	651569	651666	97
49	652246	652343	652440	652536	652633	97
50	653212	653309	653406	653502	653598	97
51	654176	654273	654369	654465	654562	96
52	655138	655234	655331	655427	655523	96
53	656098	656194	656290	656386	656481	96
54	657056	657151	657247	657343	657438	95
55	658011	658107	658202	658298	658393	95
56	658965	659060	659155	659250	659346	95
57	659916	660011	660106	660201	660296	95
58	660865	660960	661055	661150	661245	94
59	661813	661907	662002	662096	662191	94

from 1, To 10000.

Num	5	6	7	8	9	Diff
424	627878	627980	628082	628185	628287	101
425	628900	629002	629104	629206	629308	102
426	629919	630021	630123	630224	630326	101
427	630936	631038	631139	631241	631342	102
428	631951	632052	632153	632255	632356	101
429	632963	633054	633165	633266	633367	101
430	633973	634074	634175	634276	634376	100
431	634981	635081	635182	635283	635383	102
432	635986	636087	636187	636287	636388	100
433	636989	637089	637189	637289	637390	100
434	637990	638090	638190	638289	638389	99
435	638988	639088	639188	639287	639387	99
436	639981	640084	640183	640283	640382	99
437	640978	641077	641177	641276	641375	99
438	641970	642069	642168	642267	642366	99
439	642959	643058	643156	643255	643354	99
440	643946	644044	644143	644242	644340	98
441	644931	645029	645127	645225	645324	98
442	645913	646011	646109	646208	646306	98
443	646894	646992	647089	647187	647285	98
444	647872	647969	648057	648165	648262	98
445	648848	648945	649043	649140	649237	97
446	649821	649919	650016	650113	650210	97
447	650794	650890	650987	651084	651181	97
448	651760	651859	651956	652053	652150	97
449	652630	652725	652823	652919	653016	97
450	653695	653791	653888	653984	654080	97
451	654658	654754	654850	654946	655042	96
452	655619	655714	655810	655906	656002	96
453	656577	656673	656769	656864	656960	95
454	657534	657629	657725	657820	657916	96
455	658488	658584	658679	658774	658870	95
456	659441	659536	659631	659726	659821	95
457	660391	660486	660581	660676	660771	95
458	661339	661434	661529	661623	661718	94
459	662285	662380	662474	662569	662663	94

A Table of Logarithms,

<i>Num</i>	0	1	2	3	4	<i>Diff</i>
460	662758	662852	662947	663041	663135	94
461	663701	663795	663889	663983	664078	94
462	664642	664736	664830	664924	665018	94
463	665581	665675	665768	665862	665956	94
464	666518	666612	666705	666799	666892	94
465	667453	667546	667640	667733	667826	94
466	668385	668479	668572	668665	668758	94
467	669317	669410	669503	669596	669689	93
468	670246	670339	670431	670524	670617	93
469	671173	671265	671358	671451	671543	93
470	672098	672190	672283	672375	672467	93
471	673021	673113	673205	673297	673390	92
472	673942	674034	674126	674218	674310	92
473	674861	674953	675045	675136	675228	92
474	675778	675870	675961	676053	676145	91
475	676694	676785	676876	676968	677059	91
476	677607	677698	677789	677881	677972	91
477	678518	678609	678700	678791	678882	91
478	679428	679519	679610	679700	679791	91
479	680335	680426	680517	680607	680698	91
480	681241	681332	681422	681513	681603	90
481	682145	682235	682326	682416	682506	90
482	683047	683137	683227	683317	683407	90
483	683947	684037	684127	684217	684307	90
484	684845	684935	685025	685114	685204	90
485	685742	685831	685921	686010	686100	89
486	686536	686626	686715	686804	686894	89
487	687529	687618	687707	687796	687885	89
488	688420	688509	688598	688687	688776	89
489	689309	689398	689486	689575	689664	89
490	690196	690285	690373	690462	690550	89
491	691081	691170	691258	691347	691435	88
492	691965	692053	692142	692230	692318	88
493	692847	692935	693023	693111	693199	88
494	693727	693815	693903	693991	694078	88
495	694605	694693	694781	694868	694956	88

from 1, To 10000.

Num	5	6	7	8	9	D
460	663230	663324	663418	663512	663607	9
461	664172	664266	664360	664454	664548	9
462	665112	665206	665399	665393	665487	9
463	666050	666143	666237	666331	666424	9
464	666986	667079	667173	667266	667359	9
465	667920	668013	668106	668199	668293	9
466	668852	668945	669038	669131	669224	9
467	669782	669874	669967	670060	670153	9
468	670710	670802	670895	670988	671080	9
469	671636	671728	671821	671913	672005	9
470	672560	672652	672744	672836	672929	9
471	673482	673574	673666	673758	673850	9
472	674402	674494	674586	674677	674769	9
473	675320	675412	675503	675595	675687	9
474	676236	676328	676419	676511	676602	9
475	677150	677242	677333	677424	677516	9
476	678063	678154	678245	678336	678427	9
477	678973	679064	679155	679246	679337	9
478	679882	679973	680063	680154	680245	9
479	680789	680879	680970	681060	681151	9
480	681693	681784	681874	681964	682055	9
481	682596	682686	682777	682867	682957	9
482	683497	683587	683677	683767	683857	9
483	684396	684486	684576	684666	684756	9
484	685294	685383	685473	685563	685652	9
485	686169	686279	686358	686458	686547	8
486	687083	687172	687261	687351	687440	8
487	687975	688064	688153	688242	688331	8
488	688865	688953	689042	689131	689220	8
489	689753	689841	689930	690019	690107	8
490	690639	690727	690816	690905	690993	8
491	691524	691612	691700	691789	691877	8
492	692406	692494	692583	692671	692759	8
493	693287	693375	693463	693551	693639	8
494	694166	694254	694342	694430	694517	8
495	695044	695131	695219	695307	695394	8

A Table of Logarithms,

num	0	1	2	3	4	D. 10
96	695482	695569	695657	695744	695832	87
97	696356	696444	696531	696618	696706	87
98	697229	697317	697404	697491	697578	87
99	698101	698188	698275	698362	698449	87
00	698970	699057	699144	699231	699317	87
01	699838	699924	700011	700098	700184	87
02	700704	700790	700877	700963	701050	86
03	701518	701604	701691	701777	701863	86
04	702430	702517	702603	702689	702775	86
05	703391	703477	703563	703649	703735	86
06	704151	704236	704322	704408	704494	86
07	705008	705094	705179	705265	705350	86
08	705864	705949	706035	706120	706206	85
09	706718	706803	706888	706974	707059	85
10	707570	707655	707740	707826	707911	85
11	708421	708506	708591	708676	708761	85
12	709270	709355	709440	709524	709609	85
13	710117	710202	710287	710371	710456	85
14	710963	711048	711132	711217	711301	84
15	711807	711892	711975	712060	712144	84
16	712650	712734	712818	712902	712986	84
17	713491	713575	713659	713742	713826	84
18	714330	714414	714497	714581	714665	84
19	715167	715251	715335	715418	715502	84
20	716003	716087	716170	716254	716337	83
21	716838	716921	717004	717088	717171	83
22	717671	717754	717837	717920	718003	83
23	718502	718585	718668	718751	718834	83
24	719331	719414	719497	719580	719663	83
25	720159	720242	720325	720407	720490	83
26	720986	721068	721151	721233	721316	82
27	721811	721893	721975	722058	722140	82
28	722634	722716	722798	722881	722963	82
29	723456	723538	723620	723702	723784	82
30	724276	724358	724440	724522	724603	82
31	725094	725176	725258	725340	725422	82

from 1, To 10000.

Num	5	6	7	8	9	Diff
496	695919	696007	696094	696182	696269	87
497	696793	696880	696968	697055	697142	87
498	697665	697752	697839	697926	698014	87
499	698535	698622	698709	698796	698883	87
500	699404	699491	699578	699664	699751	87
501	700271	700358	700444	700531	700617	87
502	701136	701222	701309	701395	701482	86
503	701999	702086	702172	702258	702344	86
504	702861	702947	703033	703119	703205	86
505	703721	703807	703893	703979	704065	86
506	704570	704665	704751	704837	704922	86
507	705436	705522	705607	705693	705778	86
508	706291	706376	706462	706547	706632	85
509	707144	707229	707315	707400	707485	85
510	707996	708081	708166	708251	708336	85
511	708846	708931	709015	709100	709185	85
512	709694	709779	709863	709948	710033	85
513	710540	710625	710710	710794	710879	85
514	711385	711470	711554	711639	711723	84
515	712229	712313	712397	712481	712566	84
516	713070	713154	713238	713323	713406	84
517	713910	713994	714078	714162	714246	84
518	714749	714833	714916	715000	715084	84
519	715586	715669	715753	715836	715920	84
520	716421	716504	716588	716671	716754	83
521	717254	717338	717421	717504	717587	83
522	718086	718169	718253	718336	718419	83
523	718917	719000	719083	719165	719248	83
524	719745	719828	719911	719994	720077	83
525	720573	720655	720738	720821	720903	83
526	721398	721481	721563	721646	721728	82
527	722222	722305	722387	722469	722552	82
528	723045	723127	723209	723291	723374	82
529	723866	723948	724030	724112	724194	82
530	724685	724767	724849	724931	725013	82
531	725503	725585	725667	725748	725830	82

A Table of Logarithms,

Num	0	1	2	3	4	Diff
568	754348	754425	754501	754578	754654	76
569	755112	755189	755255	755341	755417	76
570	755875	755951	756027	756103	756179	76
571	756636	756712	756788	756864	756940	76
572	757396	757472	757548	757624	757700	76
573	758155	758230	758306	758382	758458	76
574	758912	758988	759063	759139	759214	76
575	759668	759743	759819	759894	759970	75
576	760422	760498	760573	760649	760724	75
577	761176	761251	761326	761402	761477	75
578	761928	762003	762078	762153	762228	75
579	762679	762754	762829	762904	762978	75
580	763428	763503	763578	763653	763727	75
581	764176	764251	764326	764400	764475	75
582	764923	764998	765072	765147	765221	75
583	765669	765743	765818	765892	765966	74
584	766413	766487	766562	766635	766710	74
585	767156	767230	767304	767379	767453	74
586	767898	767972	768046	768120	768194	74
587	768638	768712	768786	768860	768934	74
588	769377	769451	769525	769599	769673	74
589	770115	770189	770263	770336	770410	74
590	770825	770926	770999	771073	771146	74
591	771587	771661	771734	771808	771881	73
592	772322	772395	772468	772542	772615	73
593	773055	773128	773201	773274	773348	73
594	773786	773860	773933	774006	774079	73
595	774517	774590	774663	774736	774809	73
596	775246	775319	775392	775465	775538	73
597	775974	776047	776120	776193	776265	73
598	776701	776774	776846	776919	776992	73
599	777427	777499	777572	777644	777717	72
600	778151	778224	778296	778368	778441	72
601	778874	778947	779019	779091	779163	72
602	779596	779669	779741	779813	779885	72
603	780317	780389	780461	780533	780605	72

from 1, To 10000.

Num	5	6	7	8	9	Diff
568	754730	754807	754883	754960	755036	76
569	756494	755570	755646	755722	755799	76
570	756256	756332	756408	756484	756560	76
571	757016	757092	757168	757244	757320	76
572	757775	757851	757927	758003	758079	76
573	758533	758609	758685	758761	758836	76
574	759290	759366	759441	759517	759592	76
575	760045	760121	760196	760272	760347	75
576	760799	760875	760950	761025	761101	75
577	761552	761627	761702	761778	761853	75
578	762303	762378	762453	762529	762604	75
579	763053	763128	763203	763278	763353	75
580	763802	763877	763952	764027	764101	75
581	764550	764624	764699	764774	764848	75
582	765296	765370	765445	765519	765594	75
583	766041	766115	766190	766264	766338	74
584	766784	766859	766933	767007	767082	74
585	767527	767601	767675	767749	767823	74
586	768268	768342	768416	768490	768564	74
587	769008	769082	769156	769230	769303	74
588	769746	769820	769894	769968	770042	74
589	770484	770557	770631	770705	770778	74
590	771220	771293	771367	771440	771514	74
591	771955	772028	772102	772175	772248	73
592	772688	772762	772835	772908	772981	73
593	773421	773494	773567	773640	773713	73
594	774152	774225	774298	774371	774444	73
595	774882	774955	775028	775100	775173	73
596	775610	775683	775756	775829	775902	73
597	776338	776411	776483	776556	776629	73
598	777064	777137	777209	777282	777354	73
599	777789	777862	777934	778006	778079	72
600	778513	778585	778658	778730	778802	72
601	779236	779308	779380	779452	779524	72
602	779957	780029	780101	780173	780245	72
603	780977	780749	780821	780893	780965	72

A Table of Logarithms,

Num	0	1	2	3	4	Diff
604	781037	781109	781181	781253	781324	72
605	781795	781827	781899	781971	782042	72
606	782473	782544	782616	782688	782759	72
607	783189	783260	783332	783403	783475	71
608	783904	783975	784046	784118	784189	71
609	784517	784689	784760	784831	784903	71
610	785330	785401	785472	785543	785615	71
611	786041	786112	786183	786254	786325	71
612	786751	786822	786893	786964	787035	71
613	787460	787531	787602	787673	787744	71
614	788168	788239	788310	788381	788451	71
615	788875	788946	789016	789087	789157	71
616	789581	789651	789722	789792	789863	70
617	790285	790356	790426	790496	790567	70
618	790988	791059	791129	791199	791269	70
619	791691	791761	791831	791901	791971	70
620	792392	792462	792532	792602	792672	70
621	793092	793162	793231	793301	793371	70
622	793790	793860	793930	794000	794070	70
623	794488	794558	794627	794697	794767	70
624	795185	795254	795324	795393	795463	70
625	795880	795949	796019	796088	796158	69
626	796574	796644	796713	796782	796852	69
627	797268	797337	797406	797475	797545	69
628	797960	798029	798098	798167	798236	69
629	798651	798720	798789	798858	798927	69
630	799341	799409	799478	799547	799616	69
631	800029	800098	800167	800236	800305	69
632	800717	800786	800854	800923	800992	69
633	801404	801472	801541	801610	801678	69
634	802089	802158	802226	802295	802363	68
635	802774	802842	802910	802979	803047	68
636	803457	803525	803594	803662	803730	68
637	804139	804208	804276	804344	804412	68
638	804821	804889	804957	805025	805093	68
639	805501	805569	805637	805705	805773	68

from 1, To 10000.

Num	5	6	7	8	9	Diff
604	781396	781468	781540	781612	781684	72
605	782114	782186	782258	782329	782401	72
606	782831	782902	782974	783046	783117	72
607	783546	783618	783689	783761	783832	71
608	784261	784332	784403	784475	784546	71
609	784974	785045	785116	785187	785259	71
610	785686	785757	785828	785899	785970	71
611	786396	786467	786538	786609	786680	71
612	787106	787177	787248	787319	787390	71
613	787815	787885	787956	788027	788098	71
614	788522	788593	788663	788734	788804	71
615	789228	789299	789369	789440	789510	71
616	789933	790004	790074	790144	790215	70
617	790637	790707	790778	790848	790918	70
618	791340	791410	791480	791550	791620	70
619	792041	792111	792181	792252	792322	70
620	792742	792812	792882	792952	793022	70
621	793441	793511	793581	793651	793721	70
622	794139	794209	794279	794349	794418	70
623	794836	794906	794976	795045	795115	70
624	795532	795602	795672	795741	795810	70
625	796227	796297	796366	796436	796505	69
626	796921	796990	797060	797129	797198	69
627	797614	797683	797752	797821	797890	69
628	798305	798374	798443	798513	798582	69
629	798996	799065	799134	799203	799272	69
630	799685	799754	799823	799892	799961	69
631	800373	800442	800511	800580	800648	69
632	801061	801129	801198	801266	801335	69
633	801747	801815	801884	801952	802021	69
634	802432	802500	802568	802637	802705	68
635	803116	803184	803252	803321	803389	68
636	803798	803867	803935	804003	804071	68
637	804480	804548	804616	804685	804753	68
638	805161	805229	805297	805365	805433	68
639	805840	805908	805976	806044	806112	68

A Table of Logarithms,

<i>Num.</i>	0	1	2	3	4	<i>Diff</i>
640	806180	806248	806316	806384	806451	68
641	806858	806926	806993	807061	807129	68
642	807535	807603	807670	807738	807805	68
643	808211	808279	808346	808414	808481	67
644	808886	808953	809021	809088	809156	67
645	809560	809627	809694	809762	809829	67
646	810233	810300	810367	810434	810501	67
647	810904	810971	811039	811106	811173	67
648	811575	811642	811709	811776	811843	67
649	812245	812312	812379	812445	812512	67
650	812913	812980	813047	813114	813181	66
651	813581	813648	813714	813781	813848	66
652	814248	814314	814381	814447	814514	66
653	814913	814980	815046	815113	815179	66
654	815578	815644	815711	815777	815843	66
655	816241	816308	816374	816440	816506	66
656	816904	816970	817036	817102	817169	66
657	817565	817631	817698	817764	817830	66
658	818226	818292	818358	818424	818499	66
659	818885	818951	819017	819083	819149	66
660	819544	819610	819676	819741	819807	66
661	820202	820267	820333	820399	820464	66
662	820858	820924	820989	821055	821120	66
663	821514	821579	821645	821710	821776	65
664	822168	822233	822299	822364	822430	65
665	822822	822887	822952	823018	823083	65
666	823474	823539	823605	823670	823735	65
667	824126	824191	824256	824321	824386	65
668	824777	824842	824907	824972	825036	65
669	825426	825491	825556	825621	825686	65
670	826075	826140	826204	826269	826334	65
671	826723	826787	826852	826917	826981	65
672	827369	827434	827499	827563	827628	65
673	828015	828080	828144	828209	828273	54
674	828660	828724	828789	828853	828918	64
675	829304	829368	829432	829497	829561	64

from 1, To 10000.

Num	5	6	7	8	9	Diff
640	806519	806587	806655	806722	806790	68
641	807197	807264	807332	807400	807467	68
642	807873	807941	808008	808076	808143	68
643	808548	808616	808684	808751	808818	67
644	809223	809290	809358	809425	809492	67
645	809896	809964	810031	810098	810165	67
646	810569	810636	810703	810770	810837	67
647	811240	811307	811374	811441	811508	67
648	811910	811977	812044	812111	812178	67
649	812579	812646	812713	812780	812847	67
650	813247	813314	813381	813448	813514	66
651	813914	813981	814048	814114	814181	66
652	814581	814647	814714	814780	814847	66
653	815246	815312	815379	815445	815511	66
654	815910	815976	816042	816109	816175	66
655	816573	816639	816705	816771	816838	66
656	817235	817301	817367	817433	817499	66
657	817896	817962	818028	818094	818160	66
658	818556	818622	818688	818754	818819	66
659	819215	819281	819347	819412	819478	66
660	819873	819939	820004	820070	820136	66
661	820530	820596	820661	820727	820792	66
662	821186	821251	821317	821382	821448	66
663	821841	821906	821972	822037	822103	65
664	822495	822560	822626	822691	822756	65
665	823148	823213	823279	823344	823409	65
666	823800	823865	823930	823996	824061	65
667	824451	824516	824581	824646	824711	65
668	825101	825166	825231	825296	825361	65
669	825751	825815	825880	825945	826010	65
670	826399	826464	826528	826593	826658	65
671	827046	827111	827175	827240	827305	65
672	827692	827757	827821	827886	827951	65
673	828338	828402	828466	828531	828596	64
674	828982	829046	829111	829175	829240	64
675	829625	829690	829754	829818	829882	64

A Table of Logarithms,

<i>Num</i>	0	1	2	3	4	<i>Diff</i>
576	829947	830011	830075	830139	830204	64
577	830589	830653	830717	830781	830845	64
578	831230	831294	831358	831422	831486	64
579	831870	831934	831998	832062	832126	64
580	832509	832573	832637	832700	832764	64
581	833147	833211	833275	833338	833402	64
582	833784	833848	833912	833975	834039	64
583	834421	834484	834548	834611	834675	64
584	835056	835120	835183	835247	835310	63
585	835691	835754	835817	835881	835944	63
586	836324	836387	836451	836514	836577	63
587	836957	837020	837083	837146	837209	63
588	837588	837652	837715	837778	837841	63
589	838219	838282	838345	838408	838471	63
590	838849	838912	838975	839038	839101	63
591	839478	839541	839604	839667	839729	63
592	840106	840169	840232	840294	840357	63
593	840733	840796	840859	840921	840984	63
594	841359	841422	841485	841547	841610	63
595	841985	842047	842110	842172	842235	62
596	842609	842672	842734	842796	842859	62
597	843233	843295	843357	843420	843482	62
598	843855	843918	843980	844042	844104	62
599	844477	844539	844601	844664	844726	62
700	845098	845160	845222	845284	845346	62
701	845718	845780	845842	845904	845966	62
702	846337	846399	846461	846523	846585	62
703	846955	847017	847079	847141	847202	62
704	847573	847634	847696	847758	847819	62
705	848189	848251	848312	848374	848435	62
706	848805	848866	848928	848989	849051	61
707	849419	849481	849542	849604	849665	61
708	850033	850095	850156	850217	850279	61
709	850646	850707	850769	850830	850891	61
710	851258	851319	851381	851442	851503	61
711	851869	851931	851992	852053	852114	61

from 1, To 10000.

Num	5	6	7	8	9	Diff
676	830268	830332	830396	830460	830525	64
677	830909	830973	831037	831102	831166	64
678	831550	831614	831678	831742	831806	64
679	832189	832253	832317	832381	832445	64
680	832828	832892	832956	833019	833083	64
681	833466	833530	833593	833657	833721	64
682	834103	834166	834230	834294	834357	64
683	834738	834802	834866	834929	834993	64
684	835373	835437	835500	835564	835627	63
685	836007	836071	836134	836197	836261	63
686	836641	836704	836767	836830	836894	63
687	837273	837336	837399	837462	837525	63
688	837904	837969	838030	838093	838156	63
689	838534	838597	838660	838723	838786	63
690	839164	839237	839289	839352	839415	63
691	839792	839855	839918	839981	840043	63
692	840420	840482	840545	840608	840671	63
693	841046	841109	841172	841234	841297	63
694	841672	841735	841797	841860	841922	63
695	842297	842360	842422	842484	842547	62
696	842921	842983	843046	843108	843170	62
697	843544	843606	843669	843731	843793	62
698	844166	844229	844291	844353	844415	62
699	844788	844850	844912	844974	845036	62
700	845408	845470	845532	845594	845656	62
701	846028	846090	846151	846213	846275	62
702	846646	846708	846770	846832	846894	62
703	847264	847326	847388	847449	847511	62
704	847881	847943	848004	848066	848127	62
705	848497	848559	848620	848682	848743	62
706	849112	849174	849235	849297	849358	61
707	849726	849788	849849	849911	849972	61
708	850340	850401	850462	850524	850585	61
709	850952	851014	851075	851136	851197	61
710	851564	851625	851686	851747	851809	61
711	852175	852236	852297	852358	852419	61

A Table of Logarithms,

Num	0	1	2	3	4	Diff
712	852480	852541	852602	852663	852724	61
713	853089	853150	853211	853272	853333	61
714	843698	853759	853820	853881	853941	61
715	854306	854367	854428	854488	854549	61
716	854913	854974	855034	855095	855156	61
717	855519	855580	855640	855701	855761	61
718	856124	856185	856245	856306	856366	60
719	856729	856789	856850	856910	856970	60
720	857333	857393	857453	857513	857574	60
721	857935	857995	858056	858116	858176	60
722	858537	858597	858657	858718	858778	60
723	859138	859198	859258	859318	859378	60
724	859739	859799	859859	859918	859978	60
725	860338	860398	860458	860518	860578	60
726	860937	860996	861056	861116	861176	60
727	861534	861594	861654	861714	861773	60
728	862131	862191	862251	862310	862370	60
729	862727	862787	862847	862906	862966	60
730	863323	863382	863442	863501	863561	59
731	863917	863977	864036	864096	864155	59
732	864511	864570	864630	864689	864748	59
733	865104	865163	865223	865282	865341	59
734	865696	865755	865814	865873	865933	59
735	866287	866346	866405	866465	866524	59
736	866878	866937	866996	867055	867114	59
737	867467	867526	867585	867644	867703	59
738	868056	868115	868174	868233	868292	59
739	868644	868703	868762	868821	868879	59
740	869232	869290	869349	869408	869466	59
741	869818	869877	869935	869994	870053	59
742	870404	870462	870521	870579	870638	58
743	870989	871047	871106	871164	871223	58
744	871573	871631	871690	871748	871806	58
745	872156	872215	872273	872331	872389	58
746	872739	872797	872855	872913	872972	58
747	873321	873379	873437	873495	873553	58

from 1, To 10000.

Num	5	6	7	8	9	Diff
712	852785	852846	852907	852968	853029	61
713	853394	853455	853516	853576	853637	61
714	854002	854063	854124	854184	854245	61
715	854610	854670	854731	854792	854852	61
716	855216	855277	855337	855398	855459	61
717	855822	855882	855943	856003	856064	61
718	856427	856487	856548	856608	856668	60
719	857031	857091	857151	857212	857272	60
720	857634	857694	857754	857815	857875	60
721	858236	858296	858357	858417	858477	60
722	858838	858898	858958	859018	859078	60
723	859439	859499	859559	859619	859679	60
724	860038	860098	860158	860218	860278	60
725	860637	860697	860757	860817	860877	60
726	861236	861295	861355	861415	861475	60
727	861833	861893	861952	862012	862072	60
728	862430	862489	862549	862608	862668	60
729	863025	863085	863144	863204	863263	60
730	863620	863680	863739	863798	863858	59
731	864214	864274	864333	864392	864452	59
732	864808	864867	864926	864985	865045	59
733	865400	865459	865518	865578	865637	59
734	865992	866051	866110	866169	866228	59
735	866583	866642	866701	866760	866819	59
736	867173	867232	867291	867350	867409	59
737	867762	867821	867880	867939	867997	59
738	868350	868409	868468	868527	868586	59
739	868938	868997	869056	869114	869173	59
740	869525	869584	869642	869701	869760	59
741	870111	870170	870228	870287	870345	59
742	870696	870755	870813	870872	870930	58
743	871281	871339	871398	871456	871515	58
744	871865	871923	871981	872040	872098	58
745	872448	872506	872564	872622	872681	58
746	873030	873088	873146	873204	873262	58
747	873611	873669	873727	873785	873843	58

A Table of Logarithms,

<i>Num</i>	<i>0</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>Diff</i>
748	873902	873969	874018	874076	874134	58
749	874482	874540	874598	874656	874714	58
750	875061	875119	875177	875235	875293	58
751	875640	875698	875756	875813	875871	58
752	876218	876276	876333	876391	876449	58
753	876795	876853	876910	876968	877026	58
754	877371	877429	877487	877544	877602	58
755	877947	878004	878062	878119	878177	57
756	878522	878579	878637	878694	878751	57
757	879096	879153	879211	879268	879325	57
758	879669	879726	879784	879841	879898	57
759	880242	880299	880356	880413	880471	57
760	880814	880871	880928	880985	881042	57
761	881385	881442	881499	881556	881613	57
762	881955	882012	882069	882126	882183	57
763	882524	882581	882638	882695	882752	57
764	883093	883150	883207	883264	883321	57
765	883661	883718	883775	883832	883888	57
766	884229	884285	884342	884399	884455	57
767	884795	884852	884909	884965	885022	57
768	885361	885418	885474	885531	885587	57
769	885926	885983	886039	886096	886152	56
770	886491	886547	886604	886660	886716	56
771	887054	887111	887167	887223	887280	56
772	887617	887674	887730	887786	887842	56
773	888179	888236	888292	888348	888404	56
774	888741	888797	888853	888909	888965	56
775	889302	889358	889414	889470	889526	56
776	889862	889918	889974	890030	890086	56
777	890421	890477	890533	890589	890644	56
778	890980	891035	891091	891147	891203	56
779	891537	891593	891649	891705	891760	56
780	892095	892150	892206	892262	892317	56
781	892651	892707	892762	892818	892873	56
782	893207	893262	893318	893373	893429	56
783	893762	893817	893873	893928	893984	55

from 1, To 10000.

Num	5	6	7	8	9	Diff
748	874192	874250	874308	874366	874424	58
749	874772	874830	874887	874945	875003	58
750	875351	875409	875466	875524	875582	58
751	875929	875987	876044	876102	876160	58
752	876506	876564	876622	876680	876737	58
753	877083	877141	877198	877256	877314	58
754	877659	877717	877774	877832	877889	58
755	878234	878292	878349	878407	878464	57
756	878809	878866	878924	878981	479038	57
757	879383	879440	879497	879555	879612	57
758	879956	880013	880070	880127	880185	57
759	880528	880585	880642	880699	880756	57
760	881099	881156	881213	881270	881328	57
761	881670	881727	881784	881841	881898	57
762	882240	882297	882354	882411	882468	57
763	882809	882866	882923	882980	883036	57
764	883377	883434	883491	883548	883605	57
765	883945	884002	884059	884115	884172	57
766	884512	884569	884625	884682	884739	57
767	885078	885135	885191	885248	885305	57
768	885644	885700	885757	885813	885870	57
769	886209	886265	886321	886378	886434	56
770	886773	886829	886885	886942	886998	56
771	887336	887392	887449	887505	887561	56
772	887898	887955	888011	888067	888123	56
773	888460	888516	888573	888629	888685	56
774	889021	889077	889133	889190	889246	56
775	889582	889638	889694	889750	889806	56
776	890141	890197	890253	890309	890365	56
777	890700	890756	890812	890868	890924	56
778	891259	891314	891370	891426	891482	56
779	891816	891872	891928	891983	892039	56
780	892373	892428	892484	892540	892595	56
781	892929	892985	893040	893096	893151	56
782	893484	893540	893595	893651	893706	56
783	894039	894094	894150	894205	894261	55

A Table of Logarithms,

<i>num</i>	0	1	2	3	4	<i>Diff</i>
84	894316	894371	894427	894482	894538	55
85	894870	894925	894980	895036	895091	55
86	895422	895478	895533	895588	895643	55
87	895975	896030	896085	896140	896195	55
88	896526	896581	896636	896691	896747	55
89	897077	897132	897187	897242	897297	55
90	897627	897682	897737	897792	897847	55
91	898176	898231	898286	898341	898396	55
92	898725	898780	898835	898890	898944	55
93	899273	899328	899383	899437	899492	55
94	899820	899875	899930	899985	900039	55
95	900367	900422	900476	900531	900586	55
96	900913	900968	901022	901077	901131	55
97	901458	901513	901567	901622	901676	54
98	902003	902057	902112	902166	902220	54
99	902547	902601	902655	902710	902764	54
100	903090	903144	903198	903253	903307	54
101	903632	903687	903741	903795	903849	54
102	904174	904228	904283	904337	904391	54
103	904715	904770	904824	904878	904932	54
104	905276	905310	905364	905418	905472	54
105	905796	905850	905904	905958	906012	54
106	906335	906389	906443	906497	906550	54
107	906873	906927	906981	907035	907089	54
108	907411	907465	907519	907573	907626	54
109	907948	908002	908056	908109	908163	54
110	908485	908539	908592	908646	908699	54
111	909021	909074	909128	909181	909235	54
112	909556	909609	909663	909716	909770	53
113	910090	910144	910197	910251	910304	53
114	910624	910678	910731	910784	910838	53
115	911158	911211	911264	911317	911371	53
116	911690	911743	911797	911850	911903	53
117	912222	912275	912323	912381	912435	53
118	912753	912806	912859	912913	912966	53
119	913283	913337	913390	913443	913496	53

from 1, To 10000.

Num	5	6	7	8	9	Diff
784	894593	894648	894704	894759	894814	55
785	895146	895201	895257	895312	895367	55
786	895699	895754	895809	895864	895919	55
787	896251	896306	896361	896416	896471	55
788	896802	896857	896912	896967	897022	55
789	897352	897407	897452	897517	897572	55
790	897902	897957	898012	898067	898122	55
791	898451	898506	898561	898615	898670	55
792	898999	899054	899109	899164	899218	55
793	899547	899602	899656	899711	899766	55
794	900094	900149	900203	900258	900312	55
795	900640	900695	900749	900804	900859	55
796	901186	901240	901295	901349	901404	55
797	901731	901785	901840	901894	901948	54
798	902275	902329	902384	902438	902492	54
799	902818	902873	902927	902981	903036	54
800	903361	903416	903470	903524	903578	54
801	903903	903958	904012	904066	904120	54
802	904445	904499	904553	904607	904661	54
803	904986	905040	905094	905148	905202	54
804	905526	905580	905634	905688	905742	54
805	906065	906119	906173	906227	906281	54
806	906604	906658	906712	906766	906820	54
807	907142	907196	907250	907304	907358	54
808	907680	907734	907787	907841	907895	54
809	908217	908270	908324	908378	908431	54
810	908753	908807	908860	908914	908967	54
811	909288	909342	909395	909449	909502	54
812	909823	909877	909930	909984	910037	53
813	910358	910411	910464	910518	910571	53
814	910891	910944	910998	911051	911104	53
815	911424	911477	911530	911584	911637	53
816	911956	912009	912063	912116	912169	53
817	912488	912541	912594	912647	912700	53
818	913019	913072	913125	913178	913231	53
819	913549	913602	913655	913708	913761	52

A Table of Logarithms,

Num	0	1	2	3	4	Diff
820	913814	913867	913920	913973	914026	53
821	914343	914396	914449	914502	914555	53
822	914872	914925	914977	915030	915583	53
823	915400	915453	915505	915558	915611	53
824	915927	915980	916033	916085	916138	53
825	916454	916507	916559	916612	916664	53
826	916980	917033	917085	917138	917190	53
827	917505	917558	917611	917663	917715	53
828	918030	918083	918135	918188	918240	53
829	918554	918607	918659	918712	918764	53
830	919078	919130	919183	919235	919287	52
831	919601	919653	919706	919758	919810	52
832	920123	920175	920228	920280	920332	52
833	920645	920697	920749	920801	920853	52
834	921166	921218	921270	921322	921374	52
835	921686	921738	921790	921842	921894	52
836	922206	922258	922310	922362	922414	52
837	922725	922777	922829	922881	922933	52
838	923244	923296	923348	923399	923451	52
839	923762	923814	923865	923917	923969	52
840	924279	924331	924383	924434	924486	52
841	924796	924848	924899	924951	925002	52
842	925312	925364	925415	925467	925518	52
843	925828	925879	925931	925982	926034	51
844	926342	926394	926445	926497	926548	51
845	926857	926908	926959	927011	927062	51
846	927370	927422	927473	927524	927576	51
847	927883	927935	927986	928037	928088	51
848	928396	928447	928498	928549	928601	51
849	928908	928959	929010	929061	929112	51
850	926419	929470	929521	929572	929623	51
851	929930	929981	930032	930083	930134	51
852	930440	930491	930541	930592	930643	51
853	930949	931000	931051	931102	931153	51
854	931458	931509	931560	931610	931661	51
855	931966	932017	932068	932118	932169	51

from 1, To 10000.

Num	5	6	7	8	9	Diff
820	914079	914132	914184	914237	914290	53
821	914608	914660	914713	914766	914819	53
822	915136	915189	915241	915294	915347	53
823	915664	915716	915769	915822	915874	53
824	916191	916243	916296	916349	916401	53
825	916717	916770	916822	916875	916927	53
826	917243	917295	917348	917400	917453	53
827	917768	917820	917873	917925	917978	53
828	918292	918345	918397	918459	918502	53
829	918816	918869	918921	918973	919026	53
830	919340	919392	919444	919496	919549	52
831	919862	919914	919967	920019	920071	52
832	920384	920436	920489	920541	920593	52
833	920906	920958	921010	921062	921114	52
834	921426	921478	921530	921582	921634	52
835	921946	921998	922050	922102	922154	52
836	922466	922518	922570	922622	922674	52
837	922985	923037	923088	923140	923192	52
838	923503	923555	923607	923658	923710	52
839	924021	924072	924124	924176	924228	52
840	924538	924589	924641	924693	924744	52
841	925054	925106	925157	925209	925261	52
842	925570	925621	925673	925725	925776	52
843	926085	926137	926188	926239	926291	51
844	926600	926651	926702	926754	926805	51
845	927114	927165	927216	927268	927319	51
846	927627	927678	927730	927781	927832	51
847	928140	928191	928242	928293	928345	51
848	928652	928703	928754	928805	928856	51
849	929163	929214	929266	929317	929364	51
850	929674	929725	929776	929827	929878	51
851	930185	930236	930287	930338	930389	51
852	930694	930745	930796	930847	930898	51
853	931203	931254	931305	931356	931407	51
854	931712	931763	931814	931864	931915	51
855	932220	932271	932322	932372	932423	51

A Table of Logarithms,

	0	1	2	3	4	Diff
6	932474	932524	932575	932626	932677	51
7	932981	933031	933082	933133	933183	51
8	933487	933538	933588	933639	933690	51
9	933993	934044	934094	934145	934195	51
50	934498	934549	934599	934650	934700	50
51	935003	935054	935104	935154	935205	50
52	935507	935558	935608	935658	935709	50
53	936011	936061	936111	936162	936212	50
54	936514	936564	936614	936665	936715	50
55	937016	937066	937116	937167	937217	50
56	937518	937568	937618	937668	937718	50
57	938019	938069	938119	938169	938219	50
58	938520	938570	938620	938670	938720	50
59	939020	939070	939120	939170	939220	50
70	939519	939569	939619	939669	939719	50
71	940018	940068	940118	940168	940218	50
72	940516	940566	940616	940666	940716	50
73	941014	941064	941114	941163	941213	50
74	941511	941561	941611	941660	941710	50
75	942008	942058	942107	942157	942206	50
76	942504	942554	942603	942653	942702	50
77	943000	943049	943096	943148	943198	49
78	943494	943544	943593	943643	943692	49
79	943989	944038	944088	944137	944186	49
80	944483	944532	944581	944631	944680	49
81	944976	945025	945074	945124	945173	49
82	945469	945518	945567	945616	945665	49
83	945961	946010	946059	946108	946157	49
84	946452	946501	946551	946600	946649	49
85	946943	946992	947041	947090	947139	49
86	947434	947483	947532	947581	947630	49
87	947924	947973	948022	948070	948119	49
88	948413	948462	948511	948560	948609	49
89	948902	948951	948999	949048	949097	49
90	949390	949439	949488	949536	949585	49
91	949878	949926	949975	950024	950073	49

from 1, To 10000.

Num	5	6	7	8	9	Diff
856	932727	932776	932829	932879	932930	51
857	933234	933285	933335	933385	933437	51
858	933740	933791	933841	933892	933943	51
859	934246	934296	934347	934397	934448	51
860	934751	934801	934852	934902	934953	50
861	935255	935306	935356	935406	935457	50
862	935759	935809	935860	935910	935960	50
863	936262	936313	936363	936413	936463	50
864	936765	936815	936865	936916	936966	50
865	937267	937317	937367	937418	937468	50
866	937769	937819	937869	937919	937969	50
867	938269	938319	938370	938420	938470	50
868	938770	938820	938870	938920	938970	50
869	939270	939320	939369	939420	939470	50
870	939769	939819	939868	939918	939968	50
871	940267	940317	940367	940417	940467	50
872	940765	940815	940865	940915	940964	50
873	941263	941313	941362	941412	941462	50
874	941760	941809	941859	941909	941958	50
875	942256	942306	942355	942405	942454	50
876	942752	942801	942851	942900	942950	50
877	943247	943297	943346	943396	943445	49
878	943742	943791	943841	943890	943939	49
879	944236	944285	944335	944384	944433	49
880	944729	944779	944828	944877	944927	49
881	945222	945272	945321	945370	945419	49
882	945715	945764	945813	945862	945911	49
883	946207	946256	946305	946354	946403	49
884	946698	946747	946796	946845	946894	49
885	947189	947238	947287	947336	947385	49
886	947679	947728	947777	947826	947875	49
887	948168	948217	948266	948315	948364	49
888	948657	948706	948755	948804	948853	49
889	949146	949195	949244	949292	949341	49
890	949633	949683	949731	949780	949829	49
891	950121	950170	950219	950267	950316	49

A Table of Logarithms,

Num	0	1	2	3	4	Diff
892	950365	950414	950462	950511	950560	49
893	950851	950900	950949	950997	951046	49
894	951337	951386	951435	951483	951532	49
895	951823	951872	951920	951969	952017	49
896	952308	952356	952405	952453	952502	48
897	952792	952841	952889	952938	952986	48
898	953276	953325	953373	953421	953470	48
899	953760	953808	953856	953905	953953	48
900	954242	954291	954339	954387	954435	48
901	954725	954773	954821	954869	954918	48
902	955207	955255	955303	955351	955399	48
903	955688	955736	955784	955832	955880	48
904	956168	956216	956264	956313	956361	48
905	956649	956697	956745	956793	956840	48
906	957128	957176	957224	957272	957320	48
907	957607	957655	957703	957751	957799	48
908	958086	958134	958181	958229	958277	48
909	958564	958612	958659	958707	958755	48
910	959041	959089	959137	959184	959232	48
911	959518	959566	959614	959661	959709	48
912	959995	960042	960090	960138	960185	48
913	960471	960518	960566	960613	960661	48
914	960946	960994	961041	961089	961136	47
915	961421	961468	961516	961563	961611	47
916	961895	961943	961990	962038	962085	47
917	962369	962417	962464	962511	962559	47
918	962843	962890	962937	962985	963032	47
919	963315	963363	963410	963457	963504	47
920	963788	963835	963882	963929	963977	47
921	964260	964307	964354	964401	964448	47
922	964731	964778	964825	964872	964919	47
923	965202	965249	965296	965343	965390	47
924	965672	965719	965766	965813	965860	47
925	966142	966189	966236	966283	966329	47
926	966611	966658	966705	966752	966798	47
927	967080	967127	967173	967220	967267	47

from 1, To 10000.

Num	5	6	7	8	9	Diff
892	950608	950657	950706	950754	950803	49
893	951095	951143	951192	951240	951289	49
894	951580	951629	951677	951726	951775	49
895	952066	952114	952163	952211	952259	49
896	952550	952599	952647	952696	952744	48
897	953034	953083	953131	953180	953228	48
898	953518	953566	953615	953663	953711	58
899	954001	954049	954098	954146	954194	48
900	954484	954532	954580	954628	954677	48
901	954966	955014	955062	955110	955158	48
902	955447	955495	955543	955591	955640	48
903	955928	955976	956024	956072	956120	48
904	956409	956457	956505	956553	956601	48
905	956888	956936	956984	957032	957080	48
906	957368	957416	957464	957512	957559	48
907	957847	957894	957942	957990	958038	48
908	958325	958373	958421	958468	958516	48
909	958803	958850	958898	958946	958994	48
910	959280	959328	959375	959423	959471	48
911	959757	959804	959852	959900	959947	48
912	960233	960280	960328	960376	960423	48
913	960709	960756	960804	960851	960899	48
914	961184	961231	961279	961326	961374	47
915	961658	961706	961753	961801	961848	47
916	962132	962180	962227	962275	962322	47
917	962606	962653	962701	962748	962795	47
918	963079	963126	963174	963221	963268	47
919	963552	963599	963646	963693	963741	47
920	964024	964071	964118	964165	964212	47
921	964495	964542	964590	964637	964684	47
922	964966	965013	965061	965108	965155	47
923	965437	965484	965531	965578	965625	47
924	965907	965954	966001	966048	966095	47
925	966376	966423	966470	966517	966564	47
926	966845	966892	966939	966986	967033	47
927	967314	967361	967408	967454	967501	47

A Table of Logarithms,

Num	C	I	2	3	4	Diff
928	967548	967595	967642	967688	967735	47
929	968016	968062	968109	968156	968203	47
930	968483	968530	968576	968623	968670	47
931	968950	968996	969043	969090	969136	47
932	969416	969463	969509	969556	969602	47
933	969882	969928	969975	970021	970068	47
934	970347	970393	970440	970486	970533	46
935	970812	970858	970904	970951	970997	46
936	971276	971322	971369	971415	971461	46
937	971740	971785	971832	971879	971925	46
938	972203	972249	972295	972342	972388	46
939	972666	972712	972758	972804	972851	46
940	973128	973174	973220	973266	973313	46
941	973590	973636	973682	973728	973774	46
942	974051	974097	974143	974189	974235	46
943	974512	974558	974604	974650	974696	46
944	974972	975018	975064	975110	975156	46
945	975432	975478	975524	975570	975616	46
946	975891	975937	975983	976029	976075	46
947	976350	976396	976442	976488	976533	46
948	976808	976854	976900	976946	976992	46
949	977266	977312	977358	977403	977449	46
950	977724	977769	977815	977861	977906	46
951	978181	978226	978272	978317	978363	46
952	978637	978683	978728	978774	978819	45
953	979093	979138	979184	979230	979275	45
954	979548	979594	979639	979685	979730	45
955	980003	980049	980094	980140	980185	45
956	980458	980503	980549	980594	980640	45
957	980912	980957	981003	981048	981093	45
958	981366	981411	981456	981501	981547	45
959	981819	981864	981909	981954	982000	45
960	982271	982316	982362	982407	982452	45
961	982723	982769	982814	982859	982904	45
962	983175	983220	983265	983310	983356	45
963	983626	983671	983716	983762	983807	45

from 1, To 10000.

Num	5	6	7	8	9	Diff
928	967782	957829	967875	967922	967969	47
929	968249	968296	968343	968389	968436	47
930	968716	968763	968810	968856	968903	47
931	969183	969229	969276	969323	969369	47
932	969649	969695	969742	969789	969835	47
933	970114	970161	970207	970254	970300	47
934	970579	970626	970672	970719	970765	46
935	971044	971090	971137	971183	971229	46
936	971508	971554	971601	971647	971693	46
937	971971	972018	972064	972110	972157	46
938	972434	972481	972527	972573	972619	46
939	972897	972943	972989	973035	973082	46
940	973359	973405	973451	973497	973543	46
941	973820	973866	973913	973959	974005	46
942	974281	974327	974374	974420	974466	46
943	974742	974788	974834	974880	974926	46
944	975202	975248	975294	975340	975386	46
945	975662	975707	975753	975799	975845	46
946	976121	976167	976212	976258	976304	46
947	976579	976625	976671	976717	976763	46
948	977037	977083	977129	977175	977220	46
949	977495	977541	977586	977632	977678	46
950	977952	977998	978043	978089	978135	46
951	978409	978454	978500	978546	978591	46
952	978865	978911	978956	479002	979047	45
953	979321	979365	979412	979457	979503	45
954	979776	979821	979867	979912	979958	45
955	980231	980276	980322	980367	980412	45
956	980685	980730	980776	980821	980867	45
957	981139	981184	981229	981275	981320	45
958	981592	981637	981683	981728	981773	45
959	982045	982090	982135	982181	982226	45
960	982467	982513	982558	982603	982648	45
961	982949	982994	983040	983085	983130	45
962	983401	983446	983491	983536	983581	45
963	983852	983897	983942	983987	984032	45

A Table of Logarithms,

<i>Num</i>	0	1	2	3	4	<i>Diff</i>
964	984077	984122	984167	984212	984257	45
965	984527	984572	984617	984662	984707	45
966	984977	985022	985067	985112	985157	45
967	985426	985471	985516	985561	985606	45
968	985875	985920	985965	986010	986055	45
969	986324	986369	986413	986458	986503	45
970	986772	986816	986861	986906	986951	45
971	987219	987264	987309	987353	987398	45
972	987666	987711	987756	987800	987845	45
973	988113	988157	988202	988247	988291	45
974	988559	988604	988648	988693	988737	45
975	989005	989049	989094	989138	989183	45
976	989450	989494	989539	989583	989628	44
977	989895	989939	989983	990028	990072	44
978	990339	990383	990428	990472	990516	44
979	990783	990827	990871	990916	990960	44
980	991226	991270	991315	991359	991403	44
981	991669	991713	991757	991802	991846	44
982	992112	992155	992200	992244	992288	44
983	992554	992598	992642	992686	992730	44
984	992995	993039	993083	993127	993172	44
985	993436	993480	993524	993568	993613	44
986	993877	993921	993965	994009	994053	44
987	994317	994361	994405	994449	994493	44
988	994757	994801	994845	994889	994933	44
989	995196	995240	995284	995328	995372	44
990	995635	995679	995723	995767	995811	44
991	996074	996117	996161	996205	996249	44
992	996512	996555	996599	996643	996687	44
993	996949	996993	997037	997080	997124	44
994	997386	997430	997474	997517	997561	44
995	997823	997867	997910	997954	997998	44
996	998259	998303	998346	998390	998434	44
997	998695	998739	998782	998826	998869	44
998	999130	999174	999218	999261	999305	44
999	999565	999609	999652	999696	999739	44